Fuzzy Inventory Model For Deteriorating Items

H.P.UMAP

Department of Statistics

Yashavantrao Chavan Institute of Science

Satara.(M.S.), India.

E-mail: [umaphemant@gmail.com](mailto:umaphemant@gmail.com)

***Abstract -*  In this paper the total cost and optimum order quantity are obtained in fuzzy sense for deteriorating items and especially considering demand being dependent on selling price and frequency of advertisement. Function principle method is used for defuzzification. Also the median rule is applied to find the optimum Economic Order Quantity [EOQ] and shortage quantity. Solution procedure is illustrated by a numerical example. The sensitivity analysis of the optimum solution with respect to the changes in the different parameter values is also discussed.**

***Keywords - Fuzzy model, Crisp Model, Function principle.***

I. INTRODUCTION

Deterioration of an item plays an important role in inventory management. The commodities like food grains vegetables, fruits, chemicals etc. deteriorate during their storage period. Therefore the loss due to deterioration cannot be ignored while determining the optimal inventory policy. Certain models have been developed in the area of deteriorating inventories considering the demand rate to be constant, stock-dependent, time-dependent, ramp type or selling price dependent. However in the present competitive market, the frequency of advertisement of an item changes its demand. The advertisement of an item by well-known media such as Newspapers, Magazines, Radio, Television, and through the sales representatives motivate the people to buy more and more. Selling price is also one of the important factors in purchasing of an item. Naturally lesser selling price increases the demand whereas higher price has the reverse effect. Wee H.M.[ 6 ] used selling price dependent demand. It is more realistic to assume that the demand rate is a function of frequency of advertisements and selling price of an item simultaneously. Recently, Bhunia A.K. and Maiti M.[ 1 ] are used the demand depend upon selling price, frequency of advertisement and linear trend in time to develop inventory model for deteriorating items.

For developing the EOQ model, it is always considered that the deterioration rate, various costs in inventory control are constant in the crisp model. But in reality, it is not so certain. Hence this uncertainty should be treated as fuzzy number. Yao et. Al. [ 7,8 ] used the Centroid and Signed distance methods to defuzzify the total cost. Chen S.H. and Wang C.C. [ 2 ] consider the back order inventory model with the fuzzy yearly demand, fuzzy order cost, fuzzy inventory cost and fuzzy backorder cost. Then Function principle method is used to defuzzify the total cost. Similarly in [ 3 ,4 ], Chen S.H. used the same Function principle method but these models are also for non-deteriorating items.

Here in this paper the total cost and optimum order quantity are obtained in fuzzy sense for deteriorating items and especially considering demand being dependent on selling price and frequency of advertisement by using function principal method for defuzzification to obtain total fuzzy inventory cost. Also the median rule is applied to find the optimum Economic Order Quantity [EOQ] and shortage quantity. Solution procedure is illustrated by a numerical example. The sensitivity analysis of the optimum solution with respect to the changes in the different parameter values is also discussed.

II. ASSUMPTIONS

1. The scheduling period is constant and no lead-time.
2. Demand rate R is dependent linearly on the unit selling price and non-linearly on frequency of advertisement i.e., **** where a, b and  are non-negative constants.
3. Shortages are allowed and totally backlogged.
4. Deteriorating rate is age specific failure rate.
5. The advertisement cost is fraction of the total selling price per cycle.

III. NOTATIONS

## T : Scheduling time of one cycle.

R : Demand rate per unit time; 

 : Deterioration rate.

Q(t) : Inventory level at time t.

CH : Total Holding cost per cycle.

C1 : Holding cost per unit.

CS  : Total Shortage cost per cycle.

C2  : Shortage cost per unit.

Sd : Total deteriorating units.

CD : Total deteriorating cost per cycle.

Cd  : Deteriorating cost per unit.

CA  : Advertisement cost per cycle.

P : Selling price per unit.

N : Number of advertisements.

 : Advertisement cost 

S : Initial stock level.

S1 : Maximum shortage level.

TC : Total inventory cost per cycle.

(wavy bar (~) represents the fuzzification of the

parameters)

IV. FIGURE

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S

Time

0 t1 S1

V. MATHEMATICAL ANALYSIS

i)CRISP MODEL

The initial stock level is S at time t = 0, then inventory level decreases due to demand mainly and partially by deterioration. The stock reaches to zero level at t = t1.Then shortages occur and accumulate to the level S1 at t = T.

The differential equation describing the state of inventory in the interval (0 , t1) is given by ;  --- (1)

Solving above differential equation using boundary condition at t = 0, Q(t) = S ,We get ,

; 

--- (2)

using boundary condition at t = t1 , Q(t1) = 0 , we get

 --- (3)

The differential equation describing the state of inventory in the interval ( t1 , T) is given by ,

 ;  --- (4)

by integrating both sides and solving using condition at t = t1 , Q(t1) = 0 , we get ,

; --- (5)

using condition at t = T ,Q(t) = -S1 , we get ,

---(6)

Total deteriorating units during the time interval (0 , T) are

 ; 



Solving above integral, we get ,



Therefore the deteriorating cost is given by ,



 --- (7)

Holding cost over the time period (0 , T) is given by ,



Solving above integral using equation (2) , we get

 --- (8)

Shortage cost is given by



Solving above integral by using equation (5) , we get

 ---- (9)

Advertisement cost per cycle is





Then the total inventory cost is given by ,

TC = CH + CD + CS + CA



 --- (10)

The above equation can be simplified using series form of logarithmic term and ignoring second and higher terms as follows

1) 

2) 

3) Second and higher terms are negligible with this approximation. Assuming their validity, for all relevant expressions, we get



for minimizations second derivative should be greater than zero , we get the following condition,



Therefore total inventory cost becomes,

 ---(11)

To obtain optimum order quantity differentiating TIC partially w.r.t. S and equate to zero

 ---(12)

The optimum order level is given by,

 ---(13)

ii) FUZZY MODEL

In the above developed crisp model, it was assumed that all the parameters were fixed or could be predicted with certainty, but in real life situations, they will fluctuate little from the actual value. Therefore these parameters of model could not be assumed constant. Usually rate of deterioration is vague in nature, thus instead of considering rate of deterioration as constant the EOQ model is developed with the assumption that deterioration rate is a fuzzy number. Similarly holding cost and shortage cost are also considered as a fuzzy number.

In this model deterioration rate, holding cost and shortage cost are represented by trapezoidal fuzzy numbers. By using function principle method fuzzy total cost and fuzzy optimum ordered quantity is obtained. The equation of fuzzy total inventory cost is



where 

and , , , ,  and  are fuzzy points.

VI. DEFUZZIFICATION BY FUNCTION PRINCIPLE

In equation ( 12 ), Suppose ,  and  are fuzzy numbers with the trapezoidal membership function , such as

=( C11,C12, C13, C14 ) =( C21, C22, C23, C24 )

=( ,,, ) =( ,,,)

By using function principle, the membership function of  can be defined as

=(, , ,  ), where



For i=1,2,3,4 , By using median rule, above equation can be revised as





To obtain optimum order quantity differentiating TICm partially w.r.t. S and equate to zero 



Solving the above equation, the optimum order level is given by,



**VII. Numerical Example**

**1) Crisp model**

**Input :**

a=100, b=0.5, P=4, N=2, =0.3 , C1=0.5 , C2=5, Cd=4 , T=1 ,  =0.05, =0.05,

**Output :**

S=86.18, t1=0.70, S1=35.97, TC=81.10

**2) Fuzzy Model**

SENSITIVITY ANALYSIS

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **N** |  | **C1** | **C2** |  |  | **S** | **S1** | **t1** | **TC** |
| 1 | 0.3  0.3  0.4  0.4 | (.3,.4,.5,.6)  (.4,.5,.6,.7)  (.3,.4,.5,.6)  (.4,.5,.6,.7) | (3,4,5,6)  (4,5,6,7)  (3,4,5,6)  (4,5,6,7) | (.01,.03,.05,.07)  (.03,.05,.07,.09)  (.01,.03,.05,.07)  (.03,.05,.07,.09) | (.03,.04,.05,.06)  (.04,.05,.06,.07)  (.03,.04,.05,.06)  (.04,.05,.06,.07) | 74.20  73.36  74.20  73.36 | 24.90  26.24  24.90  26.24 | 0.74  0.73  0.75  0.73 | 60.23  75.83  60.22  75.83 |
| 2 | 0.3  0.3  0.4  0.4 | (.3,.4,.5,.6)  (.4,.5,.6,.7)  (.3,.4,.5,.6)  (.4,.5,.6,.7) | (3,4,5,6)  (4,5,6,7)  (3,4,5,6)  (4,5,6,7) | (.01,.03,.05,.07)  (.03,.05,.07,.09)  (.01,.03,.05,.07)  (.03,.05,.07,.09) | (.03,.04,.05,.06)  (.04,.05,.06,.07)  (.03,.04,.05,.06)  (.04,.05,.06,.07) | 87.77  86.88  94.06  93.11 | 34.14  35.60  36.59  38.15 | 0.72  0.70  0.72  0.70 | 89.79  111.99  96.23  120.03 |

VIII. CONCLUDING REMARK

In sensitivity analysis of fuzzy model with advertisement, the optimum values are presented along with the combination of two values of number of orders (N) and two intervals of values of fuzzy parameters. Decision maker can select the optimum results of any one suitable case. It is expected that if expenditure on advertisement increases, then total inventory cost increases and it becomes in observation. Also it is seen that, due to increase in the values of various parameters the total inventory cost in fuzzy model increases. So the control on the values of these parameters is necessary.

IX. FUTURE SCOPE

As discussed earlier the function principle method is not considered in many fuzzy inventory models for defuzzification. The models are lack of deterioration of an items, so it is necessary to develop the inventory models for deteriorating items with the above method. Here we have used the function principle method for with-shortages inventory model of deteriorating items. This model can be extended with finite replenishment. The proposed model can also be developed for multi-item, multi-objective inventory model.

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