An Intuitionistic Fuzzy Approach to Solve Multi-objective Transportation Problem

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Abstract.
The aim of this paper is to develop a new concept of optimization technique using Atanassov's intuitionistic fuzzy sets (IFS), to solve multi-objective transportation problem. The idea is based on extension of fuzzy optimization. In this paper, we used a special type of linear membership and non-membership function have been taken for the degree of rejection of objectives and the degree of satisfaction to solve the multi-objective transportation problem. Find out the optimal compromise solutions. In addition, numerical example is also presented to illustrate the methodology.

Keywords: Multi-objective, Transportation Problem, Linear membership function, Non-linear membership function, Intuitionistic Fuzzy.

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1. Introduction:-

Fuzziness in transportation problem was investigated and applied by various researchers ([2], [8], [11], [12]). However in many cases they do not represent exactly the real problems. In practical situation, due to insufficiency in the information available, it is not easy to describe the constraint conditions by ordinary fuzzy sets and the evaluation of membership values is not always possible up to decision makers(DM)'s satisfaction, consequently there remains an indeterministic part of which hesitation survives. The fuzzy set uses only a membership function to indicate the degree of belongingness to the fuzzy set under consideration. The degree of non-belongingness is just automatically the compliment of 1. However, a human being who expresses the degree of membership of a given element in a fuzzy set very often does not express corresponding degree of non-membership as the complement to 1. Sometimes it seems to be more natural to describe imprecise and uncertain opinions not only by membership functions. However, in some situations players also describe their negative feelings, i.e., degrees of dissatisfaction about the outcomes of the game. On the other hand, the players can only estimate their aspiration levels (goals) and/or their values with some imprecision. But it is possible that he/she is not so sure about it. In other words, there may be hesitation about the approximate payoff values. It is therefore most likely that the players have some indeterminacy or hesitation about these approximations. Fuzzy set theory is thus not enough to model the optimization problems involving indeterminacy. Therefore it is reasonable to believe that there is some indeterminacy in estimating the aspiration levels. In such situation intuitionistic fuzzy(IF) set, introduced by Atanassov [6, 7], serve better our required purpose. IF set is characterized by two functions expressing the degree of membership and the degree of non-membership, respectively. The hesitation degree is equal to 1 minus both the degree of membership and the degree of non-membership. The IF set may express and describe information more abundant and flexible than the fuzzy sets when uncertain information is involved. The IF set has been applied to different areas such as decision making problem [3], medical diagnosis [13] etc.

In this paper, intuitionistic fuzzy optimization (IFO) is used to solve transportation problems. The advantage of the IFO technique is that it gives the richest apparatus for formulation of optimization problems because this method can consider together the degree of acceptance and the degree of rejection. We assume that each manufacturer has the strategy to obtain profit. We assume that, DMs want to optimize the degree of attainment of the IF objectives. The degree of membership of a solution may be defined as the degree of acceptance and the degree of non-membership of a solution as the degree of rejection. The sum of degrees of acceptance and rejection is considered as less than or equal to 1.

In this paper, we choose linear membership function and non-membership function to establish IF environment.

Intuitionistic Fuzzy Sets:-

The IFS introduced by Atanassov [2, 3] is characterized by two functions expressing the degree of belongingness and the degree of non-belongingness respectively.

Definitions:-

Let $U = \{x_1, x_2, \cdots, x_n\}$ be a finite universal set. An IFS $\tilde{A}$ in a given universal set U is an object having the form

$$\tilde{A} = \left\{ \left\langle x, \mu_{\tilde{A}}(x), \upsilon_{\tilde{A}}(x) \right\rangle / x \in U \right\}$$

Where, the functions $\mu_{\underset{A}{\sim}}(x): U \to [0,1]$ *and* $\upsilon_{\underset{A}{\sim}}(x): U \to [0,1]$

define the degree of membership and non-membership of an element $x \in U$ to the set $A \subseteq U$ respectively, such that they satisfy the following condition:

$$0 \le \mu_{\underset{A}{\sim}}(x) + \upsilon_{\underset{A}{\sim}}(x) \le 1, \forall\ x \in U$$

Which is known as IF condition. The degree of acceptance and of non-acceptance can be arbitrary. The set of all IFSs over U is defined by IFS (U).

2. **Intuitionistic fuzzy optimization model:- Definition**

A Multi-objective transportation problem may be stated mathematically as:

$$\text{Minimize } \tilde{Z}_p = \begin{cases} \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{1} x_{ij} \\ \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{2} x_{ij} \\ \vdots \\ \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{p} x_{ij} \end{cases} \qquad (1)$$

Subject to

$$\sum_{j=1}^{n} X_{ij} = a_i, \qquad i=1,2,...,m \qquad (2)$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \qquad j=1,2,...,n \qquad (3)$$

$$X_{ij} \ge 0 \quad \forall \quad i, j \qquad (4)$$

Where the subscript on Z_p and superscript on c_{ij}^{p} denote the P^{th} penalty criterion; $a_i > 0\ \forall\ i,\ b_j > 0\ \forall\ j,\ c_{ij}^{p}\ \forall\ i,j$ and

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad \textbf{(Balanced condition)}$$

IFO, a method of uncertainty optimization, is put forward on the basis of fuzzy set. It is an extension of fuzzy optimization in which the degrees of rejection of objective(s) and constraints are considered together with the degrees of satisfaction ([11]. According to IFO theory, we are to maximize the degree of acceptance and to minimize the degree of rejection of IF objective(s) and constraints as follows:

$$\max_{X \in R^n} \{\mu_k(x)\}; \qquad \min_{X \in R^n} \{\upsilon_k(x)\}$$

$$\mu_k(x),\ \upsilon_k(x) \geq 0; \qquad \mu_k(x) \geq \upsilon_k(x)$$

$$0 \leq \mu_{\tilde{A}}(x) + \upsilon_{\tilde{A}}(x) \leq 1; \qquad K = 1, 2, 3, \ldots, n$$

Where, $\mu_k(x)$ denotes the degree of acceptance and $\upsilon_k(x)$ denotes the degree of rejection of x from the kth IFS. The formula can be transformed to the following system

$$\max \lambda,\ \min \mu \qquad \lambda \leq \mu_K(x);\ \mu \geq \nu_K(x)$$

$$\lambda \geq \mu; \qquad \lambda + \mu \leq 1; \qquad \lambda, \mu \geq 0$$

$$k = 1, 2, 3, \ldots., n$$

Where, λ denotes the minimal acceptable degree of objective(s) and constraints μ denotes the maximal degree of rejection of objective(s) and constraints. The IFO model can be changed into the following crisp optimization model as:

$$\max(\lambda - \mu); \quad \lambda \leq \mu_K(x); \quad \mu \geq \nu_K(x)$$

$$\lambda \geq \mu; \qquad \lambda + \mu \leq 1; \qquad \lambda, \mu \geq 0$$

$$k = 1, 2, 3, \ldots., n$$

which can be easily solved by various mathematical programming.

3. Fuzzy Algorithm to solve multi-objective multi-index transportation problem

Step 1:

Solve the Multi-objective multi-index transportation problem as a single objective transportation problem P times by taking one of the objectives at a time

Step 2:

From the results of step 1, determine the corresponding values for every objective at each solution derived. According to each solution and value for every objective, we can find pay-off matrix as follows

$$\begin{array}{c} \\ X^{(1)} \\ X^{(2)} \\ \vdots \\ X^{(P)} \end{array} \begin{array}{c} \begin{matrix} Z_1(X) & Z_2(X) & \ldots & Z_p(X) \end{matrix} \\ \begin{bmatrix} Z_{11} & Z_{12} & \ldots & Z_{1p} \\ Z_{21} & Z_{22} & \ldots & Z_{2p} \\ \ldots & \ldots & \ldots & \ldots \\ Z_{p1} & Z_{p2} & \ldots & Z_{pp} \end{bmatrix} \end{array}$$

Where, $X^{(1)},X^{(2)},...,X^{(p)}$ are the isolated optimal solutions of the P different transportation problems for P different objective functions $Z_{ij}=Z_j(X^i) \quad (i=1,2,...,p \quad \& \quad j=1,2,...,p)$ be the i-th row and j-th column element of the pay-off matrix.

Step 3:

From step 2, we find for each objective the worst (U_p) and the best (L_p) values corresponding to the set of solutions, where,

$$U_p = \max(Z_{1p}, Z_{2p}, ..., Z_{pp}) \quad \text{and} \quad L_p = Z_{pp} \qquad p=1,2,...,P$$

An initial fuzzy model of the problem (1)-(4)

can be stated as

Find X_{ij} $\quad i=1,2,...,m \quad j=1,2,...,n,$

so as to satisfy

$$Z_p \lesssim L_p \qquad p=1,2,...,P \tag{5}$$

subject to

$$\sum_{j=1}^{n} X_{ij} = a_i, \qquad i=1,2,...,m \tag{6}$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \qquad j=1,2,...,n \tag{7}$$

$$X_{ij} \geq 0 \qquad \forall \quad i, j \tag{8}$$

Step 4: Case (i)

Define linear membership function for the p-th objective function as follows:

$$\mu_p(X) = \begin{cases} 1 & \text{if } Z_p(X) \leq L_p \\ \dfrac{U_p - Z_p(X)}{U_p - L_p} & \text{if } L_p < Z_p < U_p \\ 0 & \text{if } U_p \leq Z_p \end{cases} \tag{9}$$

$$\nu_p(X) = \begin{cases} 0 & \text{if } Z_p(X) \leq L_p \\ \dfrac{Z_p(X) - L_p}{U_p - L_p} & \text{if } L_p < Z_p < U_p \\ 1 & \text{if } L_p \leq Z_p \end{cases}$$

Where, p is small change in L_p. L_p and U_p are the lower and upper bounds of $Z_p(X)$. Also, $0 < p < U_p - L_p$, for each p = 1, 2, 3, …,k

Now, the above IFTP can be transferred to the following crisp optimization problem which can be easily solved numerically by programming technique (MATLAB)

Step 5:

Find an equivalent crisp model by using a linear membership function for the initial fuzzy model

$$\text{Maximize} \quad \lambda\text{-}\mu \tag{10}$$

subject to

$$\lambda \le \frac{U_p - \tilde{Z}_p(X)}{U_p - L_p} \tag{11}$$

$$\mu \ge \frac{\tilde{Z}_p(X) - L_p}{U_p - L_p}$$

$$\sum_{j=1}^{n} X_{ij} = a_i, \qquad i=1,2,...,m \tag{12}$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \qquad j=1,2,...,n \tag{13}$$

$$X_{ij} \ge 0 \qquad \forall \quad i, j \quad \text{and} \quad \lambda \ge 0 \tag{14}$$

Step 6: Solve the crisp model by an appropriate mathematical programming algorithm.

In the general IFTP total number of supplies need not be equal to total number of demands and the equality relation in the constraints are also not crisp. That is in the above IFTP (1 to 4) suppose the constraints $\sum_{j=1}^{n} X_{ij} \approx a_i$ and $\sum_{i=1}^{m} X_{ij} \approx b_j$ are also lying in the IF set. Giving the appropriate violations in the constrains we can transform it into the crisp problem as given below

$$\text{Maximize} \quad \lambda - \mu \tag{15}$$

subject to

$$\lambda \le \frac{U_p - \tilde{Z}_p(X)}{U_p - L_p} \tag{16}$$

$$\mu \ge \frac{\tilde{Z}_p(X) - L_p}{U_p - L_p}$$

$$\sum_{j=1}^{n} X_{ij} = a_i, \qquad i=1,2,...,m \tag{17}$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \qquad j=1,2,...,n \tag{18}$$

$$X_{ij} \ge 0 \qquad \forall \quad i, j \quad \text{and} \quad \lambda \ge 0 \;\; \& \; \lambda \ge \mu, \quad \lambda + \mu \le 1 \tag{19}$$

4. **Numerical Example**

Consider the following transportation problem. Goods from three sources have to be delivered to three destinations. Cost of delivery from sources to the respective markets (in thousand rupees) is given in each cell. The demands of goods in sources and supply of goods in each source are given in the last row and column respectively. An optimal transportation plan which minimizes the costs and loss has to be determined

Cost table

	Destination-1	Destination-2	Destination-3	Supply
Source-1	16	19	12	14
Source-2	22	13	19	16
Source-3	4	28	8	12
Demand	10	15	17	

Loss table

	Destination-1	Destination-2	Destination-3	Supply
Source-1	9	14	12	14
Source-2	16	10	14	16
Source-3	8	20	6	12
Demand	10	15	17	

Minimize $Z_1 = 16X_{11} + 19X_{12} + 12X_{13} + 22_{21} + 13X_{22} + 19X_{23} + 4X_{31} + 28X_{32} + 8X_{33}$ (20)

Minimize $Z_2 = 9X_{11} + 14X_{12} + 12X_{13} + 16_{21} + 10X_{22} + 14X_{23} + 8X_{31} + 20X_{32} + 6X_{33}$ (21)

$\sum_{j=1}^{3} X_{1j} = 14 \quad ; \quad \sum_{j=1}^{3} X_{2j} = 16 \quad ; \quad \sum_{j=1}^{3} X_{3j} = 12$ (22)

$\sum_{i=1}^{3} X_{i1} = 10 \quad ; \quad \sum_{i=1}^{3} X_{i2} = 15 \quad ; \quad \sum_{i=1}^{3} X_{i3} = 17$ (23)

$X_{ij} \geq 0 \quad i=1,2,3. \quad , \quad j=1,2,3.$ (24)

For, objective Z_1, we find the optimal solution as

$X^{(1)} = \{X_{11}=9 ; X_{13}=5 ; X_{21}=1 , X_{22}=15 , X_{33}=12\}$
$Z_1 = 517$

For, objective Z_2, we find the optimal solution as

$X^{(2)} = \{X_{11}=10 ; X_{13}=4 ; X_{22}=15 , X_{23}=1 ; X_{33}=12\}$
$Z_2 = 374$

Now for $X^{(1)}$ we can find out Z_2, $\quad Z_2(X^{(1)}) = 379$

Now for $X^{(2)}$ we can find out Z_1 $\quad Z_1(X^{(2)}) = 518$

Pay-off matrix is

$$\begin{array}{c} \\ X^{(1)} \\ X^{(2)} \end{array} \begin{array}{c} \begin{matrix} Z_1 & \quad Z_2 \end{matrix} \\ \begin{bmatrix} 517 & 379 \\ 518 & 374 \end{bmatrix} \end{array}$$

From this matrix $U_1 = 518, \quad U_2 = 379, \quad L_1 = 517, \quad L_2 = 374$

Find $\{X_{ij} \;, i=1,2,3 , \; j=1,2,3\}$, So as to satisfy $Z_1 \leq 517$ and $Z_2 \leq 374$,

Define membership function for the objective functions $Z_1(X)$ and $Z_2(X)$ respectively

$$\mu_1(X)=\begin{cases} 1, & \text{if} \quad Z_1(X)\le 517 \\ \dfrac{518\text{-}Z_1(X)}{518\text{-}517}, & \text{if} \quad 517< Z_1(X)<518 \\ 0, & \text{if} \quad Z_1(X)\ge 518 \end{cases}$$

$$\nu_1(X)=\begin{cases} 0, & \text{if} \quad Z_1(X)\le 517 \\ \dfrac{Z_1(X)\text{-}517}{518\text{-}517}, & \text{if} \quad 517< Z_1(X)<518 \\ 1, & \text{if} \quad 517\le Z_1(X) \end{cases}$$

$$\mu_2(X)=\begin{cases} 1, & \text{if} \quad Z_2(X)\le 374 \\ \dfrac{379\text{-}Z_2(X)}{379\text{-}374}, & \text{if} \quad 374< Z_2(X)<379 \\ 0, & \text{if} \quad Z_2(X)\ge 379 \end{cases}$$

$$\nu_2(X)=\begin{cases} 0, & \text{if} \quad Z_2(X)\le 374 \\ \dfrac{Z_2(X)\text{-}374}{379\text{-}374}, & \text{if} \quad 374< Z_2(X)<379 \\ 1, & \text{if} \quad 374\le Z_2(X) \end{cases}$$

Find an equivalent crisp model

[i] Using Fuzzy method

$$\text{Maximize} \quad \lambda, \qquad \lambda+Z_1(X)\le 518 \quad \text{and} \quad 5\lambda+Z_2(X)\le 379$$

Solve the crisp model by using an appropriate mathematical algorithm.

$$16X_{11}+19X_{12}+12X_{13}+22X_{21}+13X_{22}+19X_{23}+14X_{31}+28X_{32}+8X_{33}+\lambda\le 518$$

$$9X_{11}+14X_{12}+12X_{13}+16X_{21}+10X_{22}+14X_{23}+8X_{31}+20X_{32}+6X_{33}+5\lambda\le 379$$

Subject to (2.27)-(2.29)

The optimal compromise solution of the problem is represented as

$$X^*=\left\{\begin{matrix} x_{11}=9.5, & x_{13}=4.5, & x_{21}=0.5, & x_{22}=15, & x_{23}=0.5, & x_{33}=12 \\ \text{and rest all } x_{ij} \text{ are zeros} \end{matrix}\right\}$$

$Z_1^*=152+54+11+195+9.5+96=517.5$

and

$Z_2^*=85.5+54+8+150+7+72=376.5$

Total Cost = 894

$\boxed{\lambda=0.5}$

[ii] Using Intuitionistic Fuzzy method

$$\text{Maximize } (\lambda\text{-}\mu)$$

$$\lambda+Z_1(X)\leq 518,\ 5\lambda+Z_2(X)\leq 379,\ Z_1(X)\text{-}\mu\leq 517 \text{ and } Z_2(X)\text{-}5\mu\leq 374$$

Solve the crisp model by using an appropriate mathematical algorithm.

$$\text{Maximize } (\lambda\text{-}\mu)$$

$$16X_{11}+19X_{12}+12X_{13}+22X_{21}+13X_{22}+19X_{23}+14X_{31}+28X_{32}+8X_{33}+\lambda\leq 518$$

$$9X_{11}+14X_{12}+12X_{13}+16X_{21}+10X_{22}+14X_{23}+8X_{31}+20X_{32}+6X_{33}+5\lambda\leq 379$$

$$16X_{11}+19X_{12}+12X_{13}+22X_{21}+13X_{22}+19X_{23}+14X_{31}+28X_{32}+8X_{33}-\mu\leq 517$$

$$9X_{11}+14X_{12}+12X_{13}+16X_{21}+10X_{22}+14X_{23}+8X_{31}+20X_{32}+6X_{33}\text{-}5\mu\leq 374$$

Subject to (22)-(24)

The optimal compromise solution of the problem is represented as

$$X^*=\begin{Bmatrix} x_{11}=10, & x_{13}=4, & x_{22}=15, & x_{23}=1, & x_{33}=12 \\ \text{and rest all } x_{ij} \text{ are zeros} & & & & \end{Bmatrix}$$

$Z_1^*=160+48+195+19+96=518$

and

$Z_2^*=90+48+150+14+72=374$

Total Cost = 892

Conclusion:

In this paper, linear membership function has been used to solve the multi-objective transportation problem. Introduced non-membership function gives a general method for intuitionistic fuzzy optimization with higher degree of satisfaction. This concept allows one to define a degree of rejection which cannot be complement of the degree of acceptance. Solution obtained from IFO problems can satisfy the objective functions with higher degree than the solutions of fuzzy problems

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