Analytical solutions of fractional Huxley equation by residual power series method

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Abstract

This paper investigates the approximate solution of nonlinear Huxley equation using new analytic technique. The solution was calculated in the form of a convergent power series with easily computable components. The proposed method obtains Taylor expansion of the solution and reproduces the exact solution when the solution is polynomial.

Keywords Huxley equation, residual power series method, Taylor expansion.

1- Introduction

Generalized Burgers-Huxley equation [1-5] is a nonlinear partial differential equation of the form

$$\frac{\partial}{\partial t}u + \alpha\, u^{\delta}\,\frac{\partial}{\partial x}u - \frac{\partial^2}{\partial x^2}u = \beta u\left(1-u^{\delta}\right)\left(u^{\delta}-\gamma\right),\ 0 \le x \le 1,\ t \ge 0 \qquad (1.1)$$

where α, β, γ *and* δ *are* parameters, $\beta \ge 0, \delta > 0,\ \gamma \in (0,1)$

When $\alpha = 0,\ \delta = 1$ *equation* (1.1) reduces to the Huxley equation.

The generalized Huxley equation [6, 7] is a nonlinear partial differential equation of second order of the form

$$\frac{\partial}{\partial t}u - \frac{\partial^2}{\partial x^2}u = \beta u\left(1-u^{\delta}\right)\left(u^{\delta}-\gamma\right) \qquad (1.2)$$

with the initial condition of $u(x,0) = [\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(\sigma\gamma x)]^{\frac{1}{\delta}}$

describes nerve pulse propagation in nerve fibres and wall motion in liquid crystals. The exact solution of this equation was derived by Wang et al. [8], using nonlinear transformations and is given by

$$u(x,t) = \{\frac{\gamma}{2} + \frac{\gamma}{2}\tanh[\sigma\gamma(x + (\frac{(1+\delta-\gamma)\rho}{2(1+\delta)})t)]\}^{\frac{1}{\delta}} \qquad (1.3)$$

where $\sigma = \dfrac{\delta\rho}{4(1+\delta)}$ and $\rho = \sqrt{4\beta(1+\delta)}$

Many researchers have used various numerical methods to solve Equation (1.2) numerically, such as adomian decomposition method [1,2,3,6,7,9], spectral collocation method [2], the tanh-coth method [3], homotopy perturbation method [6], Exp-Function method [7], variational iteration method [10] and Differential Quadrature method [11] have been used in attempting to solve the Burgers-Huxley and the Huxley equations. The solitary wave solutions of the generalized Burgers-Huxley equation have been studied by the learned researchers Wang et al. [8] and El-Danaf [12]

the time-fractional Huxley equation

$$D_t^{\alpha} u - \frac{\partial^2}{\partial x^2} u = \beta u(1-u)(u-\gamma) \tag{1.4}$$

where $x \in R$, $0 < \alpha \leq 1$, $\beta \neq 0$, $\gamma \in (0,1)$

solved by the Adomian decomposition method ,the variation iteration method [13] and homotopy analysis method [14] .In this paper we Solve equation (1.4) by a new analytical methods namely Residual power series method (RPSM) . The RPS method was developed by the first author [15] as an efficient method for determining values of coefficients of the power series solution for first and the second-order fuzzy differentia equations. It has been successfully applied in the numerical solution of the generalized Lane-Emden equation, which is a highly nonlinear singular differential equation [16] and in the numerical solution of higher-order regular differential equations [17]. The RPS method is an effective and easy to construct power series solution for strongly linear and nonlinear equations without linearization, perturbation, or discretization [15–17]. Different from the classical power series method, the RPS method does not need to compare the coefficients of the corresponding terms and a recursion relation is not required. This method computes the coefficients of the power series by a chain of equations of one or more variables.

2. Applications

In this section we solve the time-fractional Huxley equation by the Residual power series method

Consider the time-fractional Huxley equation

$$D_t^{\alpha} u - \frac{\partial^2}{\partial x^2} u = \beta u(1-u)(u-\gamma) \tag{2.1}$$

where $t > 0$, $x \in R$, $0 < \alpha \leq 1$, $\beta \neq 0$, $\gamma \in (0,1)$

Equation (2.1) can be written as

$$D_t^{\alpha} u = \frac{\partial^2}{\partial x^2} u(x,t) - \beta u^3(x,t) + \beta(\gamma+1)u^2(x,t) - \beta\gamma u(x,t) \tag{2.2}$$

subject to the initial

Condition $u(x,0) = a + a\tanh[abx],$ (2.3)

Where $a = \frac{\gamma}{2},\ b = \delta\sqrt{\frac{\beta}{\gamma+1}},\ \beta \neq 0,\ \delta \neq 0,\ \gamma \in (0,1)$

The RPS method proposes the solution for $equation$ as a fractional PS about the initial point t= 0

$$u(x,t) = \sum_{n=0}^{\infty} f_n(\mathrm{x}) \frac{t^{n\alpha}}{\Gamma(1+\mathrm{n}\alpha)},\ 0 < \alpha \leq 1,\ \ x \in I,\ \ 0 \leq t < R \tag{2.4}$$

next we let $u_k(x,t)$ to denote the k-th truncated series of u(x,t)

$$u_k(x,t) = \sum_{n=0}^{\infty} f_n(\mathrm{x}) \frac{t^{n\alpha}}{\Gamma(1+\mathrm{n}\alpha)},\ 0 < \alpha \leq 1,\ \ x \in I,\ \ 0 \leq t < R \tag{2.5}$$

The 0-th RPS approximate solution of u(x,t) is

$$u_0(x,t) = u(x,0) = f(x) = a + a\tanh[abx] \tag{2.6}$$

Equation (2.5) can be written as $u_k(x,t) = f(x) + \sum_{n=1}^{\infty} f_n(\mathrm{x}) \frac{t^{n\alpha}}{\Gamma(1+\mathrm{n}\alpha)},$
$0 < \alpha \leq 1,\ \ x \in I,\ \ 0 \leq t < R$, k=1,2,3,………

we define the residual function for Equation (2.2)

$$\mathrm{Res_u}(\mathrm{x},\mathrm{t}) = \mathrm{D}_\mathrm{t}^{\alpha} u(x,t) - \frac{\partial^2}{\partial x^2} u(x,t) + \beta u^3(x,\mathrm{t}) - \beta(\gamma+1)u^2(x,\mathrm{t}) + \beta\gamma u(x,t) \tag{2.7}$$

and,therefore,the K-th residual function $Res_{u,k}$ is

$$Res_{u,k}(x,t) = \mathrm{D}_\mathrm{t}^{\alpha} u_k(x,t) - \frac{\partial^2}{\partial x^2} u_k(x,t) + \beta {u_k}^3(x,\mathrm{t}) - \beta(\gamma+1){u_k}^2(x,\mathrm{t}) + \beta\gamma u_k(x,t), \tag{2.8}$$

$$\mathrm{D}_\mathrm{t}^{(\mathrm{k}-1)\alpha} Res_{u,k}(x,0) = 0,\ 0 < \alpha \leq 1,\ \ x \in I\ ,\ \mathrm{k} = 1,2,3, \ldots \tag{2.9}$$

To determine $f_1(x)$,we consider (k=1) in Equation (2.8)

$$Res_{u,1}(x,t) = \mathrm{D}_\mathrm{t}^{\alpha} u_1(x,t) - \frac{\partial^2}{\partial x^2} u_1(x,t) + \beta {u_1}^3(x,\mathrm{t}) - \beta(\gamma+1){u_1}^2(x,\mathrm{t}) + \beta\gamma u_1(x,t),$$

But $u_1(x,t) = f(x) + \mathrm{f}_1(\mathrm{x}) \frac{t^{\alpha}}{\Gamma(1+\alpha)}$

$$Res_{u,1}(x,t) = \mathrm{D}_\mathrm{t}^{\alpha}[f(x) + \mathrm{f}_1(\mathrm{x}) \frac{t^{\alpha}}{\Gamma(1+\alpha)}] - \frac{\partial^2}{\partial x^2}[f(x) + \mathrm{f}_1(\mathrm{x}) \frac{t^{\alpha}}{\Gamma(1+\alpha)}] + \beta\left[f(x) + \mathrm{f}_1(\mathrm{x}) \frac{t^{\alpha}}{\Gamma(1+\alpha)}\right]^3 - \beta(\gamma+1)\left[f(x) + \mathrm{f}_1(\mathrm{x}) \frac{t^{\alpha}}{\Gamma(1+\alpha)}\right]^2 + \beta\gamma[f(x) + \mathrm{f}_1(\mathrm{x}) \frac{t^{\alpha}}{\Gamma(1+\alpha)}]$$

$$Res_{u,1}(x,t) = f_1(x) - f^{\backslash\backslash}(x) + \beta f^3(x) - \beta(\gamma+1)f^2(x) + \beta\gamma f(x) + \left[f_1^{\backslash\backslash}(x) + 3\beta f^2(x) f_1(x) - 2\beta(\gamma+1) f(x) f_1(x) + \beta\gamma f_1(x)\right] \frac{t^\alpha}{\Gamma(1+\alpha)} + [3\beta f(x) f_1^2(x) - \beta(\gamma+1) f_1^2(x)] \frac{t^{2\alpha}}{\Gamma(1+\alpha)^2} + \beta f_1^3(x) \frac{t^{3\alpha}}{\Gamma(1+\alpha)^3}$$

From Equation(2.9), we deduce that $Res_{u,1}(x,0) = 0$ and thus,

$$0 = f_1(x) - f^{\backslash\backslash}(x) + \beta f^3(x) - \beta(\gamma+1)f^2(x) + \beta\gamma f(x)$$

therefore,

$$f_1(x) = f^{\backslash\backslash}(x) - \beta f^3(x) + \beta(\gamma+1)f^2(x) - \beta\gamma f(x)$$

$$f_1(x) = -a^3\beta \tanh[abx]^3 + [-3a^3\beta + a^2\beta(\gamma+1)] \tanh[abx]^2 + [-3a^3\beta + 2a^2\beta(\gamma+1) - a\beta\gamma - 2a^3b^2 \operatorname{sech}[abx]^2] \tanh[abx] + [-a^3\beta + a^2\beta(\gamma+1) - a\beta\gamma]$$

To obtain $f_2(x)$ we substitute the 2nd truncated series $u_2(x,t) = f(x) + \mathrm{f}_1(\mathrm{x}) \frac{t^\alpha}{\Gamma(1+\alpha)} + f_2(x) \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}$ into the 2nd residual function $Res_{u,2}(x,t)$

$$Res_{u,2}(x,t) = \mathrm{D}_t^\alpha\left[f(x) + \mathrm{f}_1(\mathrm{x}) \frac{t^\alpha}{\Gamma(1+\alpha)} + \mathrm{f}_2(\mathrm{x}) \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right] - \frac{\partial^2}{\partial x^2}\left[f(x) + \mathrm{f}_1(\mathrm{x}) \frac{t^\alpha}{\Gamma(1+\alpha)} + \mathrm{f}_2(\mathrm{x}) \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right] + \beta \left[f(x) + \mathrm{f}_1(\mathrm{x}) \frac{t^\alpha}{\Gamma(1+\alpha)} + \mathrm{f}_2(\mathrm{x}) \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right]^3 - \beta(\gamma+1) \left[f(x) + \mathrm{f}_1(\mathrm{x}) \frac{t^\alpha}{\Gamma(1+\alpha)} + \mathrm{f}_2(\mathrm{x}) \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right]^2 + \beta\gamma\left[f(x) + \mathrm{f}_1(\mathrm{x}) \frac{t^\alpha}{\Gamma(1+\alpha)} + \mathrm{f}_2(\mathrm{x}) \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right]$$

$$Res_{u,2}(x,t) = \left[\mathrm{f}_1(\mathrm{x}) - f^{\backslash\backslash}(x) + \beta f^3(x) - \beta(\gamma+1)f^2(x) + \beta\gamma f(x)\right] + \left[\mathrm{f}_2(\mathrm{x}) - f_1^{\backslash\backslash}(x) - 2\beta(\gamma+1) f(x) f_1(x) + \beta\gamma f_1(x) + 3\beta f^2(x) f_1(x)\right] \frac{t^\alpha}{\Gamma(1+\alpha)} + [-f_2^{\backslash\backslash}(x) + 3\beta f^2(x)\mathrm{f}_2(\mathrm{x}) - 2\beta(\gamma+1) f(x)\mathrm{f}_2(\mathrm{x}) + \beta\gamma\mathrm{f}_2(\mathrm{x})] \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + [3\beta f(x) f_1^2(x) - \beta(\gamma+1) f_1^2(x)] \frac{t^{2\alpha}}{\Gamma(1+\alpha)^2} + [\beta f_1^3(x)] \frac{t^{3\alpha}}{\Gamma(1+\alpha)^3} + [6\beta f(x)\mathrm{f}_1(\mathrm{x})\mathrm{f}_2(\mathrm{x}) - 2\beta(\gamma+1) f_1(x)\mathrm{f}_2(\mathrm{x})] \frac{t^{3\alpha}}{\Gamma(1+\alpha)\Gamma(1+2\alpha)} + [3\beta f(x) f_2^2(x) - \beta(\gamma+1) f_2^2(x)] \frac{t^{4\alpha}}{\Gamma(1+2\alpha)^2} + [3\beta f_1^2(x)\mathrm{f}_2(\mathrm{x})] \frac{t^{4\alpha}}{\Gamma(1+\alpha)^2\Gamma(1+2\alpha)} + [3\beta\mathrm{f}_1(\mathrm{x}) f_2^2(x)] \frac{t^{5\alpha}}{\Gamma(1+\alpha)\Gamma(1+2\alpha)^2} + [\beta f_2^3(x)] \frac{t^{6\alpha}}{\Gamma(1+2\alpha)^3}$$

Applying D_t^α on both sides

$D_t^{\alpha} Res_{u,2}(x,t) = \left[f_2(x) - f_1^{\backslash\backslash}(x) - 2\beta(\gamma+1)f(x)f_1(x) + \beta\gamma f_1(x) + 3\beta f^2(x)f_1(x)\right] + [-f_2^{\backslash\backslash}(x) + 3\beta f^2(x)f_2(x) - 2\beta(\gamma+1)f(x)f_2(x) + \beta\gamma f_2(x)]\frac{t^{\alpha}}{\Gamma(1+\alpha)} + [3\beta f(x) f_1^2(x) - \beta(\gamma+1) f_1^2(x)]\frac{\Gamma(1+2\alpha)t^{\alpha}}{\Gamma(1+\alpha)^3} + [\beta f_1^3(x)]\frac{\Gamma(1+3\alpha)t^{2\alpha}}{\Gamma(1+2\alpha)\Gamma(1+\alpha)^3} + [6\beta f(x)f_1(x)f_2(x) - 2\beta(\gamma+1)f_1(x)f_2(x)]\frac{\Gamma(1+3\alpha)t^{2\alpha}}{\Gamma(1+\alpha)\Gamma(1+2\alpha)^2} + [3\beta f(x) f_2^2(x) - \beta(\gamma+1) f_2^2(x)]\frac{\Gamma(1+4\alpha)t^{3\alpha}}{\Gamma(1+3\alpha)\Gamma(1+2\alpha)^2} + [3\beta f_1^2(x)f_2(x)]\frac{\Gamma(1+4\alpha)t^{3\alpha}}{\Gamma(1+3\alpha)\Gamma(1+\alpha)^2\Gamma(1+2\alpha)} + [3\beta f_1(x) f_2^2(x)]\frac{\Gamma(1+5\alpha)t^{4\alpha}}{\Gamma(1+4\alpha)\Gamma(1+2\alpha)^2\Gamma(1+\alpha)} + [\beta f_2^3(x)]\frac{\Gamma(1+6\alpha)t^{5\alpha}}{\Gamma(1+5\alpha)\Gamma(1+2\alpha)^3}$

Now depending on the result of Equation(2.9)

In the case of k=2

$$D_t^{\alpha} Res_{u,2}(x,0) = 0$$

$$0 = \left[f_2(x) - f_1^{\backslash\backslash}(x) - 2\beta(\gamma+1)f(x)f_1(x) + \beta\gamma f_1(x) + 3\beta f^2(x)f_1(x)\right]$$

we get

$$f_2(x) = \left[f_1^{\backslash\backslash}(x) + 2\beta(\gamma+1)f(x)f_1(x) - \beta\gamma f_1(x) - 3\beta f^2(x)f_1(x)\right]$$

$$f_2(x) = 3a^5\beta^2\tanh[abx]^5 + \left\{15a^5\beta^2 - 5a^4\beta^2(\gamma+1)\right\}\tanh[abx]^4 + \{30a^5\beta^2 - 20a^4\beta^2(\gamma+1) + 2a^3\beta^2(\gamma^2+4\gamma+1) + (-8a^5b^4 + 12a^5b^2\beta)\operatorname{sech}[abx]^2\}\tanh[abx]^3 + \{30a^5\beta^2 - 30a^4\beta^2(\gamma+1) + 6a^3\beta^2(\gamma^2+4\gamma+1) - 3a^2\beta^2(\gamma^2+\gamma) + (24a^5b^2\beta - 8a^4b^2\beta(\gamma+1))\operatorname{sech}[abx]^2\}\tanh[abx]^2 + \{15a^5\beta^2 - 20a^4\beta^2(\gamma+1) + 6a^3\beta^2(\gamma^2+4\gamma+1) - 6a^2\beta^2(\gamma^2+\gamma) + a\beta^2\gamma^2 + (12a^5b^2\beta - 8a^4b^2\beta(\gamma+1) + 4a^3b^2\beta\gamma)\operatorname{sech}[abx]^2 + (16a^5b^4 - 6a^5b^2\beta)\operatorname{sech}[abx]^4\}\tanh[abx] + \{3a^5\beta^2 - 5a^4\beta^2(\gamma+1) + 2a^3\beta^2(\gamma^2+4\gamma+1) - 3a^2\beta^2(\gamma^2+\gamma) + a\beta^2\gamma^2 + (2a^4b^2\beta(\gamma+1) - 6a^5b^2\beta)\operatorname{sech}[abx]^4\}$$

To obtain $f_3(x)$ we substitute the 3rd truncated series $u_3(x,t) = f(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + f_3(x)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}$

into the 3rd residual function $Res_{u,3}(x,t)$

$$Res_{u,3}(x,t) = D_t^{\alpha}[f(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + f_3(x)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}] - \frac{\partial^2}{\partial x^2}[f(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + f_3(x)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}] + \beta\left[f(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + f_3(x)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}\right]^3 - \beta(\gamma+1)\left[f(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + f_3(x)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}\right]^2 + \beta\gamma[f(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + f_3(x)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}]$$

Applying $D_t^{2\alpha}$ on both sides

$D_t^{2\alpha} Res_{u,3}(x,t) = \big[f_3(x) - f_2^{\backslash\backslash}(x) + 3\beta f^2(x) f_2(x) - 2\beta(\gamma+1) f(x) f_2(x) + \beta\gamma f_2(x)\big] + [3\beta f(x)\, f_1^2(x) - \beta(\gamma+1)\, f_1^2(x)]\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2} + \big[-f_3^{\backslash\backslash}(x) + 3\beta f^2(x) f_3(x) - 2\beta(\gamma+1) f(x) f_3(x) + \beta\gamma f_3(x)\big]\frac{t^\alpha}{\Gamma(1+\alpha)} + [\beta\, f_1^3(x)]\frac{\Gamma(1+3\alpha)}{\Gamma(1+\alpha)^4} t^\alpha + [6\beta f(x) f_1(x) f_2(x) - 2\beta(\gamma+1) f_1(x) f_2(x)]\frac{\Gamma(1+3\alpha) t^\alpha}{\Gamma(1+\alpha)^2\Gamma(1+2\alpha)} + [3\beta f(x)\, f_2^2(x) - \beta(\gamma+1)\, f_2^2(x)]\frac{\Gamma(1+4\alpha) t^{2\alpha}}{\Gamma(1+2\alpha)^3} + [3\beta\, f_1^2(x) f_2(x)]\frac{\Gamma(1+4\alpha) t^{2\alpha}}{\Gamma(1+\alpha)^2\Gamma(1+2\alpha)^2} + [6\beta f(x) f_1(x) f_3(x) - 2\beta(\gamma+1) f_1(x) f_3(x)]\frac{\Gamma(1+4\alpha) t^{2\alpha}}{\Gamma(1+\alpha)\Gamma(1+2\alpha)\Gamma(1+3\alpha)}$

$+[3\beta f_1(x)\, f_2^2(x)]\frac{\Gamma(1+5\alpha) t^{3\alpha}}{\Gamma(1+3\alpha)\Gamma(1+2\alpha)^2\Gamma(1+\alpha)} + [3\beta\, f_1^2(x) f_3(x)]\frac{\Gamma(1+5\alpha) t^{3\alpha}}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)^2} + [6\beta f(x) f_2(x) f_3(x) - 2\beta(\gamma+1) f_2(x) f_3(x)]\frac{\Gamma(1+5\alpha) t^{3\alpha}}{\Gamma(1+2\alpha)\Gamma(1+3\alpha)^2} + [\beta\, f_2^3(x)]\frac{\Gamma(1+6\alpha)\ \ t^{4\alpha}}{\Gamma(1+2\alpha)^3\Gamma(1+4\alpha)} + [3\beta\, f(x)\, f_3^2(x) - \beta(\gamma+1)\ f_3^2(x)]\frac{\Gamma(1+6\alpha)\ t^{4\alpha}}{\Gamma(1+4\alpha)\,\Gamma(1+3\alpha)^2} + [6\beta f_1(x) f_2(x) f_3(x)]\frac{\Gamma(1+6\alpha)\ t^{4\alpha}}{\Gamma(1+\alpha)\Gamma(1+2\alpha)\Gamma(1+3\alpha)\Gamma(1+4\alpha)} + [3\beta\, f_2^2(x) f_3(x)]\frac{\Gamma(1+7\alpha)\quad t^{5\alpha}}{\Gamma(1+2\alpha)^2\Gamma(1+3\alpha)\Gamma(1+5\alpha)} + [3\beta\, f_1(x)\, f_3^2(x)]\frac{\Gamma(1+7\alpha)\quad t^{5\alpha}}{\Gamma(1+\alpha)\Gamma(1+3\alpha)^2\Gamma(1+5\alpha)} + [3\beta\, f_2(x)\, f_3^2(x)]\frac{\Gamma(1+8\alpha)\quad t^{6\alpha}}{\Gamma(1+2\alpha)\Gamma(1+3\alpha)^2\Gamma(1+6\alpha)} + [\beta\, f_3^3(x)]\frac{\Gamma(1+9\alpha)\ \ t^{7\alpha}}{\Gamma(1+3\alpha)^3\Gamma(1+7\alpha)}$

Solving the equation $D_t^{2\alpha} Res_{u,3}(x,0) = 0$

We get

$$0 = \Big[f_3(x) - f_2^{\backslash\backslash}(x) + 3\beta f^2(x) f_2(x) - 2\beta(\gamma+1) f(x) f_2(x) + \beta\gamma f_2(x)] + [3\beta f(x)\, f_1^2(x) - \beta(\gamma+1)\, f_1^2(x)]\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}\Big]$$

$$f_3(x) = f_2^{\backslash\backslash}(x) - 3\beta f^2(x) f_2(x) + 2\beta(\gamma+1) f(x) f_2(x) - \beta\gamma f_2(x)] + [-3\beta f(x) + \beta(\gamma+1)]\, f_1^2(x)\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}$$

Let $f_3(x) = A(x) + B(x)$

Where $A(x) = f_2^{\backslash\backslash}(x) - 3\beta f^2(x) f_2(x) + 2\beta(\gamma+1) f(x) f_2(x) - \beta\gamma f_2(x)$

$$A(x) = f_2^{\backslash\backslash}(x) + [-3\beta f^2(x) + 2\beta(\gamma+1) f(x) - \beta\gamma] f_2(x)$$

$$B(x) = [-3\beta f(x) + \beta(\gamma+1)]\, f_1^2(x)\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}$$

$\mapsto A(X) = \{-9a^7\beta^3\}\tanh[abx]^7 + \{21a^6\beta^3(\gamma+1) - 63a^7\beta^3\}\tanh[abx]^6 + \{-189a^7\beta^3 + 126\, a^6\beta^3(\gamma+1) - a^5\beta^3(16\gamma^2 + 47\gamma + 16) + (-32a^7 b^6 +$

$72a^7b^4\beta - 66a^7b^2\beta^2)\,\mathrm{sech}[abx]^2\,\}\tanh[abx]^5 + \{-315\,a^7\,\beta^3 + 315a^6\beta^3(\gamma + 1) - a^5\beta^3(80\gamma^2 + 235\gamma + 80) + a^4\beta^3(4\gamma^3 + 34\gamma^2 + 34\gamma + 4) + (144a^7b^4\beta - 264\,a^7b^2\beta^2 - 48\,a^6b^4\beta(\gamma + 1) + 88a^6b^2\beta^2(\gamma + 1)\,\mathrm{sech}[abx]^2\}\tanh[abx]^4$
$+\{-315a^7\beta^3 + 420a^6\beta^3(\gamma + 1) - a^5\beta^3(160\gamma^2 + 470\gamma + 160) + a^4\beta^3(16\,\gamma^3 + 136\,\gamma^2 + 136\,\gamma + 16) - a^3\beta^3(8\,\gamma^3 + 23\,\gamma^2 + 8\,\gamma) + (72\,a^7b^4\beta - 396\,a^7b^2\beta^2 - 48\,a^6b^4\beta(\gamma + 1) + 264\,a^6b^2\beta^2(\gamma + 1) - a^5b^2\beta^2(28\,\gamma^2 + 104\,\gamma + 28) + 24a^5b^4\beta\gamma)\,\mathrm{sech}[abx]^2 + (416a^7b^6 - 384a^7b^4\beta + 78\,a^7b^2\beta^2\,)\mathrm{sech}[abx]^4\}\tanh[abx]^3$

$+\{-189\,a^7\beta^3 + 315\,a^6\beta^3(\gamma + 1) - a^5\beta^3(160\,\gamma^2 + 470\,\gamma + 160) + a^4\beta^3(24\,\gamma^3 + 204\,\gamma^2 + 204\,\gamma + 24) - a^3\beta^3(24\,\gamma^3 + 69\,\gamma^2 + 24\,\gamma) + 5a^2\beta^3(\,\gamma^3 + \,\gamma^2) + (-264\,a^7b^2\beta^2 + 264a^6b^2\beta^2(\gamma + 1) - a^5b^2\beta^2(56\,\gamma^2 + 208\,\gamma + 56) + 28a^4b^2\beta^2(\gamma^2 + \gamma))\,\mathrm{sech}[abx]^2 + (-528\,a^7b^4\beta + 234\,a^7b^2\beta^2 - 78a^6b^2\beta^2(\gamma + 1) + 176\,a^6b^4\beta(\gamma + 1)\,)\mathrm{sech}[abx]^4\}\tanh[abx]^2$

$+\{-63\,a^7\beta^3 + 126\,a^6\beta^3(\gamma + 1) - a^5\beta^3(80\,\gamma^2 + 235\,\gamma + 80) + a^4\beta^3(16\,\gamma^3 + 136\,\gamma^2 + 136\,\gamma + 16) - a^3\beta^3(24\,\gamma^3 + 69\,\gamma^2 + 24\,\gamma) + 10\,a^2\beta^3(\,\gamma^3 + \gamma^2) - \mathrm{a}\,\beta^3\,\gamma^2 + (-66\,a^7b^2\beta^2 + 88a^6b^2\beta^2(\gamma + 1) - a^5b^2\beta^2(28\,\gamma^2 + 104\,\gamma + 28) + 28\,a^4b^2\beta^2(\gamma^2 + \gamma) - 6a^3b^2\beta^2\,\gamma^2)\,\mathrm{sech}[abx]^2 + (-144\,a^7b^4\beta + 234\,a^7b^2\beta^2 - 156\,a^6b^2\beta^2(\gamma + 1) + 96\,a^6b^4\beta(\gamma + 1) + a^5b^2\beta^2(16\gamma^2 + 62\gamma + 16) - 48a^5b^4\beta\gamma\,)\mathrm{sech}[abx]^4 + (156a^7b^4\beta - 272a^7b^6)\,\mathrm{sech}[abx]^6\}\tanh[abx]$

$+\{-9\,a^7\beta^3 + 21\,a^6\beta^3(\gamma + 1) - a^5\beta^3(16\,\gamma^2 + 47\,\gamma + 16) + a^4\beta^3(4\,\gamma^3 + 34\,\gamma^2 + 34\,\gamma + 4) - a^3\beta^3(8\,\gamma^3 + 23\,\gamma^2 + 8\,\gamma) + 5\,a^2\beta^3(\,\gamma^3 + \,\gamma^2) - \mathrm{a}\,\beta^3\,\gamma^3 + (78\,a^7b^2\beta^2 - 78a^6b^2\beta^2(\gamma + 1) + a^5b^2\beta^2(16\gamma^2 + 62\gamma + 16) - 8\,a^4b^2\beta^2(\gamma^2 + \gamma)\,)\mathrm{sech}[abx]^4 + (72a^7b^4\beta - 24a^6b^4\beta(\gamma + 1))\,\mathrm{sech}[abx]^6\}$

$$B(x) = [-3\beta f(x) + \beta(\gamma + 1)]\, f_1^2(x)\frac{\Gamma(1 + 2\alpha)}{\Gamma(1 + \alpha)^2}$$

$\mapsto B(x) = [\{-3a^7\beta^3\}\tanh[abx]^7 +$

$\{7a^6\beta^3(\gamma + 1) - 21a^7\beta^3\}\tanh[abx]^6 + \{-63a^7\beta^3 + 42\,a^6\beta^3(\gamma + 1) - a^5\beta^3(5\gamma^2 + 16\gamma + 5) + (-12a^7b^2\beta^2)\,\mathrm{sech}[abx]^2\,\}\tanh[abx]^5 + \{-105\,a^7\,\beta^3 + 105a^6\beta^3(\gamma + 1) - a^5\beta^3(25\gamma^2 + 80\gamma + 25) + a^4\beta^3(\gamma^3 + 11\gamma^2 + 11\gamma + 1) + (-48\,a^7b^2\beta^2 + 16a^6b^2\beta^2(\gamma + 1))\,\mathrm{sech}[abx]^2\}\tanh[abx]^4$

$+\{-105a^7\beta^3 + 140a^6\beta^3(\gamma + 1) - a^5\beta^3(50\gamma^2 + 160\gamma + 50) + a^4\beta^3(4\,\gamma^3 + 44\,\gamma^2 + 44\,\gamma + 4) - a^3\beta^3(2\,\gamma^3 + 7\,\gamma^2 + 2\,\gamma) + (-72\,a^7b^2\beta^2 + 48\,a^6b^2\beta^2(\gamma + 1) - a^5b^2\beta^2(4\,\gamma^2 + 20\,\gamma + 4))\,\mathrm{sech}[abx]^2 + (-12a^7b^4\beta\,)\mathrm{sech}[abx]^4\}\tanh[abx]^3$

$+\{-63\,a^7\beta^3 + 105\,a^6\beta^3(\gamma + 1) - a^5\beta^3(50\,\gamma^2 + 160\,\gamma + 50) + a^4\beta^3(6\,\gamma^3 + 66\,\gamma^2 + 66\,\gamma + 6) - a^3\beta^3(6\,\gamma^3 + 21\,\gamma^2 + 6\,\gamma) + a^2\beta^3(\,\gamma^3 + \,\gamma^2) + (-48\,a^7b^2\beta^2 + 48a^6b^2\beta^2(\gamma + 1) - a^5b^2\beta^2(8\,\gamma^2 + 40\,\gamma + 8) + 4a^4b^2\beta^2(\gamma^2 + \gamma))\,\mathrm{sech}[abx]^2 + (-12\,a^7b^4\beta + 4\,a^6b^4\beta(\gamma + 1)\,)\mathrm{sech}[abx]^4\}\tanh[abx]^2$

$+\{-21\, a^7\beta^3+42\, a^6\beta^3(\gamma+1)-a^5\beta^3(25\,\gamma^2+80\,\gamma+25)+a^4\beta^3(4\,\gamma^3+44\,\gamma^2+44\,\gamma+4)-a^3\beta^3(6\,\gamma^3+21\,\gamma^2+6\,\gamma)+2\, a^2\beta^3(\gamma^3+\gamma^2)+(-12\, a^7b^2\beta^2+16a^6b^2\beta^2(\gamma+1)-a^5b^2\beta^2(4\,\gamma^2+20\gamma+4)+4\, a^4b^2\beta^2(\gamma^2+\gamma))\,\mathrm{sech}[abx]^2\}\tanh[abx]$

$\{-3\, a^7\beta^3+7\, a^6\beta^3(\gamma+1)-a^5\beta^3(5\,\gamma^2+16\,\gamma+5)+a^4\beta^3(\gamma^3+11\,\gamma^2+11\,\gamma+1)-a^3\beta^3(2\,\gamma^3+7\,\gamma^2+2\,\gamma)+a^2\beta^3(\gamma^3+\gamma^2)\}]\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}$

The solution in series form is given by

$$u(x,t)=f(x)+\mathrm{f}_1(\mathrm{x})\frac{t^{\alpha}}{\Gamma(1+\alpha)}+f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}+f_3(x)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}+\cdots$$

$u(x,t)=a+a\tanh[abx]+[-a^3\beta\tanh[abx]^3+\{-3a^3\beta+a^2\beta(\gamma+1)\}\tanh[abx]^2+\{-3a^3\beta+2a^2\beta(\gamma+1)-a\beta\gamma-2a^3b^2\,\mathrm{sech}[abx]^2\}\tanh[abx]\pm a^3\beta+a^2\beta(\gamma+1)-a\beta\gamma]\frac{t^{\alpha}}{\Gamma(1+\alpha)}$

$+[3a^5\beta^2\tanh[abx]^5+\{15a^5\beta^2-5a^4\beta^2(\gamma+1)\}\tanh[abx]^4+\{30a^5\beta^2-20a^4\beta^2(\gamma+1)+2a^3\beta^2(\gamma^2+4\gamma+1)+(-8a^5b^4+12a^5b^2\beta)\,\mathrm{sech}[abx]^2\}\tanh[abx]^3+\{30a^5\beta^2-30a^4\beta^2(\gamma+1)+6a^3\beta^2(\gamma^2+4\gamma+1)-3a^2\beta^2(\gamma^2+\gamma)+(24a^5b^2\beta-8a^4b^2\beta(\gamma+1))\,\mathrm{sech}[abx]^2\}\tanh[abx]^2+\{15a^5\beta^2-20a^4\beta^2(\gamma+1)+6a^3\beta^2(\gamma^2+4\gamma+1)-6a^2\beta^2(\gamma^2+\gamma)+a\beta^2\gamma^2+(12a^5b^2\beta-8a^4b^2\beta(\gamma+1)+4a^3b^2\beta\gamma)\,\mathrm{sech}[abx]^2+(16a^5b^4-6a^5b^2\beta)\,\mathrm{sech}[abx]^4\}\tanh[abx]+\{3a^5\beta^2-5\, a^4\beta^2(\gamma+1)+2a^3\beta^2(\gamma^2+4\gamma+1)-3a^2\beta^2(\gamma^2+\gamma)+a\beta^2\gamma^2+(2a^4b^2\beta(\gamma+1)-6a^5b^2\beta)\,\mathrm{sech}[abx]^4\}]\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}$

$+[\{-9a^7\beta^3\}\tanh[abx]^7+$

$\{21a^6\beta^3(\gamma+1)-63a^7\beta^3\}\tanh[abx]^6+\{-189a^7\beta^3+126\, a^6\beta^3(\gamma+1)-a^5\beta^3(16\gamma^2+47\gamma+16)+(-32a^7b^6+72a^7b^4\beta-66a^7b^2\beta^2)\,\mathrm{sech}[abx]^2\}\tanh[abx]^5+\{-315\, a^7\beta^3+315a^6\beta^3(\gamma+1)-a^5\beta^3(80\gamma^2+235\gamma+80)+a^4\beta^3(4\gamma^3+34\gamma^2+34\gamma+4)+(144a^7b^4\beta-264\, a^7b^2\beta^2-48\, a^6b^4\beta(\gamma+1)+88a^6b^2\beta^2(\gamma+1)\,\mathrm{sech}[abx]^2\}\tanh[abx]^4$

$+\{-315a^7\beta^3+420a^6\beta^3(\gamma+1)-a^5\beta^3(160\gamma^2+470\gamma+160)+a^4\beta^3(16\,\gamma^3+136\,\gamma^2+136\,\gamma+16)-a^3\beta^3(8\,\gamma^3+23\,\gamma^2+8\,\gamma)+(72\, a^7b^4\beta-396\, a^7b^2\beta^2-48\, a^6b^4\beta(\gamma+1)+264\, a^6b^2\beta^2(\gamma+1)-a^5b^2\beta^2(28\,\gamma^2+104\,\gamma+28)+24a^5b^4\beta\gamma)\,\mathrm{sech}[abx]^2+(416a^7b^6-384a^7b^4\beta+78\, a^7b^2\beta^2)\,\mathrm{sech}[abx]^4\}\tanh[abx]^3$

$+\{-189\, a^7\beta^3+315\, a^6\beta^3(\gamma+1)-a^5\beta^3(160\,\gamma^2+470\,\gamma+160)+a^4\beta^3(24\,\gamma^3+204\,\gamma^2+204\,\gamma+24)-a^3\beta^3(24\,\gamma^3+69\,\gamma^2+24\,\gamma)+5a^2\beta^3(\gamma^3+\gamma^2)+(-264\, a^7b^2\beta^2+264a^6b^2\beta^2(\gamma+1)-a^5b^2\beta^2(56\,\gamma^2+208\,\gamma+$

$56)+28a^4b^2\beta^2(\gamma^2+\gamma))\,\mathrm{sech}[abx]^2+(-528\,a^7b^4\beta+234\,a^7b^2\beta^2-78a^6b^2\beta^2(\gamma+1)+176\,a^6b^4\beta(\gamma+1))\mathrm{sech}[abx]^4\}\tanh[abx]^2$

$+\{-63\,a^7\beta^3+126\,a^6\beta^3(\gamma+1)-a^5\beta^3(80\,\gamma^2+235\,\gamma+80)+a^4\beta^3(16\,\gamma^3+136\,\gamma^2+136\,\gamma+16)-a^3\beta^3(24\,\gamma^3+69\,\gamma^2+24\,\gamma)+10\,a^2\beta^3(\gamma^3+\gamma^2)-\mathrm{a}\,\beta^3\,\gamma^2+(-66\,a^7b^2\beta^2+88a^6b^2\beta^2(\gamma+1)-a^5b^2\beta^2(28\,\gamma^2+104\,\gamma+28)+28\,a^4b^2\beta^2(\gamma^2+\gamma)-6a^3b^2\beta^2\,\gamma^2)\,\mathrm{sech}[abx]^2+(-144\,a^7b^4\beta+234\,a^7b^2\beta^2-156\,a^6b^2\beta^2(\gamma+1)+96\,a^6b^4\beta(\gamma+1)+a^5b^2\beta^2(16\gamma^2+62\gamma+16)-48a^5b^4\beta\gamma)\mathrm{sech}[abx]^4+(156a^7b^4\beta-272a^7b^6)\,\mathrm{sech}[abx]^6\}\tanh[abx]$

$+\{-9\,a^7\beta^3+21\,a^6\beta^3(\gamma+1)-a^5\beta^3(16\,\gamma^2+47\,\gamma+16)+a^4\beta^3(4\,\gamma^3+34\,\gamma^2+34\,\gamma+4)-a^3\beta^3(8\,\gamma^3+23\,\gamma^2+8\,\gamma)+5\,a^2\beta^3(\gamma^3+\gamma^2)-\mathrm{a}\,\beta^3\,\gamma^3+\left(78\,a^7b^2\beta^2-78a^6b^2\beta^2(\gamma+1)+a^5b^2\beta^2(16\gamma^2+62\gamma+16)-8\,a^4b^2\beta^2(\gamma^2+\gamma)\right)\mathrm{sech}[abx]^4+\left(72a^7b^4\beta-24a^6b^4\beta(\gamma+1)\right)\mathrm{sech}[abx]^6\}+\{-3a^7\beta^3\}\tanh[abx]^7\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}+$

$\{7a^6\beta^3(\gamma+1)-21a^7\beta^3\}\tanh[abx]^6\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}+\{-63a^7\beta^3+42\,a^6\beta^3(\gamma+1)-a^5\beta^3(5\gamma^2+16\gamma+5)+(-12a^7b^2\beta^2)\,\mathrm{sech}[abx]^2\}\tanh[abx]^5\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}+$
$\{-105\,a^7\,\beta^3+105a^6\beta^3(\gamma+1)-a^5\beta^3(25\gamma^2+80\gamma+25)+a^4\beta^3(\gamma^3+11\gamma^2+11\gamma+1)+(-48\,a^7b^2\beta^2+16a^6b^2\beta^2(\gamma+1))\,\mathrm{sech}[abx]^2\}\tanh[abx]^4\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}$

$+\{-105a^7\beta^3+140a^6\beta^3(\gamma+1)-a^5\beta^3(50\gamma^2+160\gamma+50)+a^4\beta^3(4\,\gamma^3+44\,\gamma^2+44\,\gamma+4)-a^3\beta^3(2\,\gamma^3+7\,\gamma^2+2\,\gamma)+(-72\,a^7b^2\beta^2+48\,a^6b^2\beta^2(\gamma+1)-a^5b^2\beta^2(4\,\gamma^2+20\,\gamma+4))\,\mathrm{sech}[abx]^2+(-12a^7b^4\beta)\mathrm{sech}[abx]^4\}\tanh[abx]^3\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}$

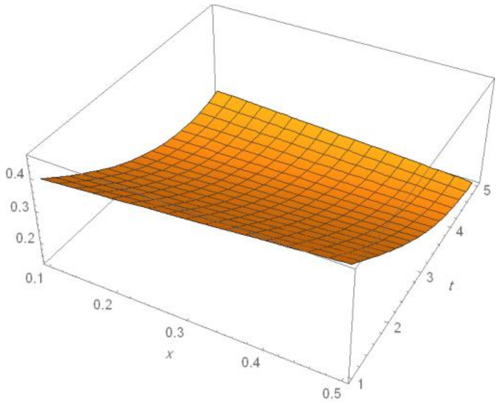
$+\{-63\,a^7\beta^3+105\,a^6\beta^3(\gamma+1)-a^5\beta^3(50\,\gamma^2+160\,\gamma+50)+a^4\beta^3(6\,\gamma^3+66\,\gamma^2+66\,\gamma+6)-a^3\beta^3(6\,\gamma^3+21\,\gamma^2+6\,\gamma)+a^2\beta^3(\gamma^3+\gamma^2)+(-48\,a^7b^2\beta^2+48a^6b^2\beta^2(\gamma+1)-a^5b^2\beta^2(8\,\gamma^2+40\,\gamma+8)+4a^4b^2\beta^2(\gamma^2+\gamma))\,\mathrm{sech}[abx]^2+(-12\,a^7b^4\beta+4\,a^6b^4\beta(\gamma+1))\mathrm{sech}[abx]^4\}\tanh[abx]^2\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}$

$+\{-21\,a^7\beta^3+42\,a^6\beta^3(\gamma+1)-a^5\beta^3(25\,\gamma^2+80\,\gamma+25)+a^4\beta^3(4\,\gamma^3+44\,\gamma^2+44\,\gamma+4)-a^3\beta^3(6\,\gamma^3+21\,\gamma^2+6\,\gamma)+2\,a^2\beta^3(\gamma^3+\gamma^2)+(-12\,a^7b^2\beta^2+16a^6b^2\beta^2(\gamma+1)-a^5b^2\beta^2(4\,\gamma^2+20\gamma+4)+4\,a^4b^2\beta^2(\gamma^2+$
$+\{-3\,a^7\beta^3+7\,a^6\beta^3(\gamma+1)-a^5\beta^3(5\,\gamma^2+\gamma))\,\mathrm{sech}[abx]^2\}\tanh[abx]\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}$
$16\,\gamma+5)+a^4\beta^3(\gamma^3+11\,\gamma^2+11\,\gamma+1)-a^3\beta^3(2\,\gamma^3+7\,\gamma^2+2\,\gamma)+a^2\beta^3(\gamma^3+\gamma^2)\}\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}]\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}$

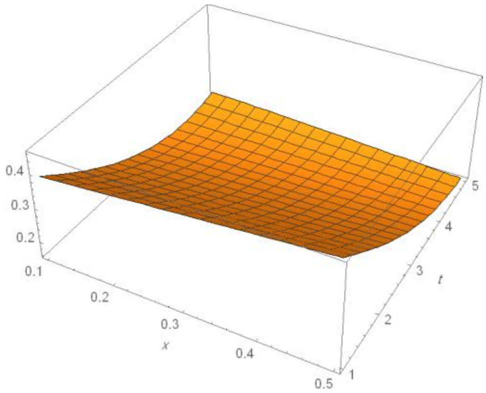
3- Numerical simulations and discussions

This section deals with the validity and effectiveness of the proposed method for Huxley equation through the different graphical representation and tabulated data.In the following we illustrate the behavior of the approximate solutions when

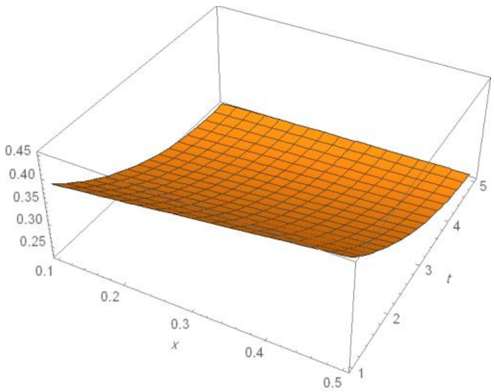
$$\boldsymbol{\gamma = 1}, \quad \boldsymbol{\beta = 1}, \quad \boldsymbol{a = \frac{1}{2}}, \quad \boldsymbol{b = \frac{1}{\sqrt{2}}}$$



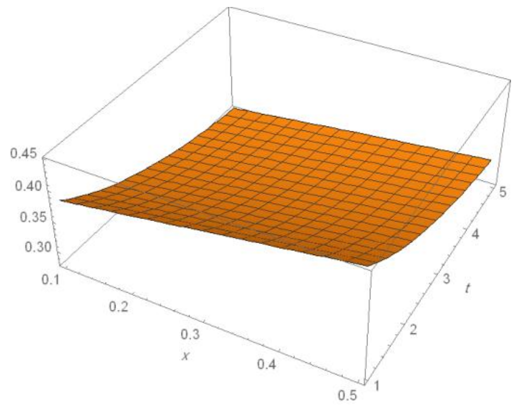
(a)



(b)



(c)



(d)

Figure (1)

(a) $\alpha = 1\,, \quad 1 \le t \le 5\,, \quad .1 \le x \le .5$

(b) $\alpha = .95\,, \quad 1 \le t \le 4\,, \quad .1 \le x \le .5$

(c) $\alpha = .75\,, \quad 1 \le t \le 4\,, \quad .1 \le x \le .5$

(d) $\alpha = .5\,, \quad 1 \le t \le 4\,, \quad .1 \le x \le .5$

Table 1: The absolute errors, $|u\_exact - u\_4|$, for Huxley equation when

$$\gamma = 1, \qquad \beta = 1, \quad a = \frac{1}{2}\,, \qquad b = \frac{1}{\sqrt{2}}, \qquad t = .1$$

x	Absolute errors using residual power series method $|u\_exact - u\_approx|$	Absolute errors using homotopy analysis method $|u\_exact - u\_approx|$	Absolute errors using Adomain decomposition method $|u\_exact - u\_approx|$	Absolute errors using variational iteration method $|u\_exact - u\_approx|$
$x = .1$	24.9636×10^{-3}	24.9648×10^{-3}	24.9636×10^{-3}	1012.46×10^{-3}
$x = .2$	24.8703×10^{-3}	24.8713×10^{-3}	24.8703×10^{-3}	1047.79×10^{-3}
$x = .3$	24.7159×10^{-3}	24.7168×10^{-3}	24.7159×10^{-3}	1083.01×10^{-3}
$x = .4$	24.5018×10^{-3}	24.5025×10^{-3}	24.5018×10^{-3}	1118.02×10^{-3}
$x = .5$	24.2302×10^{-3}	24.2308×10^{-3}	24.2302×10^{-3}	1152.74×10^{-3}

From Table 1 of absolute error, it is observed that this procedure achieves a high level of accuracy

References

[1] Hassan N.A. Ismail, Kamal Raslan, Aziza A. Abd Rabboh, " Adomian decomposition methodfor Burger's–Huxley and Burger's–Fisher equations, " Applied Mathematics and Computation 159 (2004) 291–301

[2] M. Javidi," A numerical solution of the generalized Burger's–Huxley equation by spectral collocation method, "Applied Mathematics and Computation 178 (2006) 338–344

[3] Abdul-Majid Wazwaz, " Analytic study on Burgers, Fisher, Huxley equations and combined forms of these equations," Applied Mathematics and Computation 195 (2008) 754–761

[4] Mohammad Javidi, " Spectral collocation method for the solution of the generalized Burger–Fisher equation," Applied Mathematics and Computation 174 (2006) 345–352

[5] Do_gan Kaya, Salah M. El-Sayed, " A numerical simulation and explicit solutions of the generalized Burgers–Fisher equation, " Applied Mathematics and Computation 152 (2004) 403–413

[6] S.H. Hashemi, H.R. Mohammadi Daniali, D.D. Ganji, " Numerical simulation of the generalized Huxley equation by He's homotopy perturbation method, " Applied Mathematics and Computation 192 (2007) 157–161

[7] Xin-Wei Zhou, " Exp-Function Method for Solving Huxley Equation, " Volume 2008, Article ID 538489

[8] X.Y. Wang, Z.S. Zhu, Y.K. Lu, Solitary wave solutions of the generalized Burgers–Huxley equation, Phys. Lett. A 23 (1990) 271–274

[9] I. Hashim, M.S.M. Noorani, B. Batiha, " A note on the Adomian decomposition method for the generalized Huxley equation, " Applied Mathematics and Computation 181 (2006) 1439–1445

[10] B. Batiha, M. S. M. Noorani, and I. Hashim, "Numerical simulation of the generalized Huxley equation by He's variational iteration method," *Applied Mathematics and Computation* , 186 (2007) 1322-1325

[11] Murat Sari, and Gurhan Gürarslan, "Numerical solutions of the generalized Burgers-Huxley equation by a Differential Quadrature method," *Mathematical Problems in Engineering* Volume 2009, Article ID 370765

[12] Talaat S. El-Danaf, " Solitary Wave Solutions for the Generalized Burgers-Huxley Equation," International Journal of Nonlinear Sciences and Numerical Simulation, 8(3), 315-318, 2007

[13] Talaat El-Sayed El-Danaf, " New numerical technique for solving the fractional Huxley equation," International Journal of Numerical Methods for Heat & Fluid Flow Vol. 24 No. 8, 2014 pp. 1736-1754

[14] Aref Guzali, Jalil Manafian1, Jalal Jalali, " Application of homotopy analysis method for solving nonlinear fractional partial differential equations," Asian Journal of Fuzzy and Applied Mathematics Volume 02 – Issue 02, April 2014

[15] Omar Abu Arqub, "Series solution of fuzzy differential equations under strongly generalized differentiability," Journal of Advanced Research in Applied Mathematics , Vol. 5, Issue. 1, 2013, pp. 31-52

[16] Omar Abu Arqub, Ahmad El-Ajou, A. Sami Bataineh, and I. Hashim, " A representation of the exact solution of generalized Lane-Emden equations using a new analytical method," Hindawi Publishing Corporation Abstract and Applied Analysis Volume 2013, Article ID 378593

[17] Omar Abu Arqub, Zaer Abo-Hammour, Ramzi Al-Badarneh, and ShaherMomani, " A reliable analytical method for solving higher-order initial value problems," Hindawi Publishing Corporation Discrete Dynamics in Nature and Society Volume 2013, Article ID 673829