Exact solutions for coupled nonlinear partial differential equations using G'/G method

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Abstract

(G'/G)-expansion method is examined to solve the BoitiLeonPempinelli (BLP) system and the $(2 + 1)$-dimensional breaking soliton system. The results show that this method is a powerful tool for solving systems of nonlinear PDEs., it presents exact travelling wave solutions. The obtained solutions include rational, periodical, singular, shock wave and solitary wave solutions.

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1. INTRODUCTION

Nonlinear PDEs are widely used to describe complex phenomena in various fields of sciences, such as fluid mechanics, plasma physics, astrophysics, optical fibers, solid state physics, chemical kinematics, chemical physics and geochemistry, etc., [1–12]. Studying the nonlinear waves such as soliton, breather, compacton, etc., is one of the most important problems in mathematical physics and engineering. Various mathematical methods for finding exact solutions of NLEEs have been proposed, such as tanh method [12], extended tanh method [13, 14], the symmetry method [15], sine-cosine method [16], the improved (G'/G)-expansion method [17],the $(G'/G, 1/G)$-expansion method [18], homogeneous balance [19], F-expansion method [20], generalized expansion method [21] and (G'/G) method [22–24]. Recently, In [24] (G'/G)-expansion method is applied to solve ZK and CKP equations in multicomponent plasma. (G'/G)-expansion is a direct, effective and powerful method for finding analytical solutions of nonlinear partial differential equations. In [22] Wang et al. proposed the method, while Zhang et al. [23] proposed a generalized (G'/G)-expansion method to improve and extend G'/G method to solve variable coefficient and high dimensional equations. In this work the (G'/G)-expansion method with computations are performed with computer algebra system such as Mathematica to deduce many exact breather-type solutions containing rational, periodical, singular and solitary wave solutions. In this article, many exact travelling solutions are obtained for the BoitiLeonPempinelli (BLP) equation and the $(2 + 1)$-dimensional breaking soliton equation.

The manuscript is organised in the following fashion; In Section II, The extended (G'/G)-expansion method is described. In section III we apply extended (G'/G)-expansion method to solve the (BLP) equation and the $(2 + 1)$-dimensional breaking soliton equation systems. conclusions are given in section V.

2. DESCRIPTION OF THE METHOD

For the general NLEE

$$P(u, u_x, u_t, u_y, u_{xx}, ...) = 0 \tag{1}$$

where $u = u(x, y, t)$ and P is a polynomial in u and its derivatives. We seek its solutions in the form

$$u(\zeta) = \sum_{i=0}^{n} a_i \left(\frac{G'(\zeta)}{G(\zeta)} \right)^i, \tag{2}$$

where a_i are real constants with $a_i \neq 0$ to be determined, n is a positive integer to be determined. The function $G(\zeta)$ is the solution of the auxiliary linear ordinary differential equation

$$AGG'' - BGG' - C(G')^2 - E(G)^2 = 0, \tag{3}$$

where A, B, C and E are real constants to be determined, and

$$\zeta = x + y - \lambda t \tag{4}$$

where λ is the speed of the travelling wave.

step 1. Using transformation (4) we obtain an ordinary differential equation (ODE) for $u = u(\zeta)$:

$$E(u, u', u'', u''', ...) = 0 \tag{5}$$

step 2. By balancing the highest nonlinear terms and the highest-order partial differential terms in the given NLEE we can determine n.

step 3. Substituting Eq. (2) and (3) into Eq. (5) and collecting coefficients of polynomial of $\left(\frac{G'(\zeta)}{G(\zeta)}\right)$, then setting each coefficient to zero yields a set of algebraic equations for a_i (i=0,1,2,...,n), A, B, C, E and λ.

step 4. Solving the system of algebraic equations in step 2 for a_i,A, B, C, E and λ using Maple or Mathematica.

step 5. As Eq. (2) possesses the general solutions:

Case 1. If $B \neq 0, \Psi = A - C$ and $\Omega = B^2 + 4E(A - C) > 0$, then

$$\left(\frac{G'(\zeta)}{G(\zeta)} \right) = \frac{B}{2\Psi} + \frac{\sqrt{\Omega}}{2\Psi} \left(\frac{c_1 \sinh(\frac{\sqrt{\Omega}}{2\Psi}\zeta) + c_2 \cosh(\frac{\sqrt{\Omega}}{2\Psi}\zeta)}{c_1 \cosh(\frac{\sqrt{\Omega}}{2\Psi}\zeta) + c_2 \sinh(\frac{\sqrt{\Omega}}{2\Psi}\zeta)} \right), \tag{6}$$

Case 2. If $B \neq 0, \Psi = A - C$ and $\Omega = B^2 + 4E(A-C) < 0$, then

$$\left(\frac{G'(\zeta)}{G(\zeta)}\right) = \frac{B}{2\Psi} + \frac{\sqrt{-\Omega}}{2\Psi}\left(\frac{-c_1 \sin(\frac{\sqrt{-\Omega}}{2\Psi}\zeta) + c_2 \cos(\frac{\sqrt{-\Omega}}{2\Psi}\zeta)}{c_1 \cos(\frac{\sqrt{-\Omega}}{2\Psi}\zeta) + c_2 \sin(\frac{\sqrt{-\Omega}}{2\Psi}\zeta)}\right), \tag{7}$$

Case 3. If $B \neq 0, \Psi = A - C$ and $\Omega = B^2 + 4E(A-C) = 0$, then

$$\left(\frac{G'(\zeta)}{G(\zeta)}\right) = \frac{B}{2\Psi} + \frac{\sqrt{-\Omega}}{2\Psi}\left(\frac{c_2}{c_1 + c_2\zeta}\right), \tag{8}$$

Case 4. If $B = 0, \Psi = A - C$ and $\Delta = \Psi E > 0$, then

$$\left(\frac{G'(\zeta)}{G(\zeta)}\right) = \frac{\sqrt{\Delta}}{2\Psi}\left(\frac{c_1 \sinh(\frac{\sqrt{\Delta}}{2\Psi}\zeta) + c_2 \cosh(\frac{\sqrt{\Delta}}{2\Psi}\zeta)}{c_1 \cosh(\frac{\sqrt{\Delta}}{2\Psi}\zeta) + c_2 \sinh(\frac{\sqrt{\Delta}}{2\Psi}\zeta)}\right), \tag{9}$$

Case 5. If $B = 0, \Psi = A - C$ and $\Delta = \Psi E < 0$, then

$$\left(\frac{G'(\zeta)}{G(\zeta)}\right) = \frac{\sqrt{-\Delta}}{2\Psi}\left(\frac{-c_1 \sin(\frac{\sqrt{-\Delta}}{2\Psi}\zeta) + c_2 \cos(\frac{\sqrt{-\Delta}}{2\Psi}\zeta)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2\Psi}\zeta) + c_2 \sin(\frac{\sqrt{-\Delta}}{2\Psi}\zeta)}\right), \tag{10}$$

3. APPLICATION OF THE METHOD

1. BoitiLeonPempinelli (BLP) system

Let us consider the (2+1)-dimensional coupled BoitiLeonPempinelli (BLP) system,

$$\begin{aligned} u_{ty} &= (u^2 - u_x)_{xy} + 2v_{xx}, \\ v_t &= v_{xx} + 2(uv)_x \end{aligned} \tag{11}$$

Balancing the highest derivative term with non-linear terms, hence we may assume that

$$\begin{aligned} u(x,y,t) &= k_0 + k_1\phi(\zeta) \\ v(x,y,t) &= \mu_0 + \mu_1\phi(\zeta) + \mu_2\phi^2(\zeta) \end{aligned} \tag{12}$$

where $\zeta = x + y - \lambda t$. Substituting Eq. (12) into Eq. (11) with aide of Eq. (3) and collecting coefficients of polynomial of ϕ^i and equating them to zero, we get a system of algebraic equations for k_0, k_1, μ_0, μ_1 and μ_2

$$-\frac{2Ek_1\mu_0}{A}-\frac{BE\mu_1}{A^2}-\frac{\lambda E\mu_1}{A}-\frac{2Ek_0\mu_1}{A}-\frac{2E^2\mu_2}{A^2}=0 \tag{13}$$

$$-\frac{2Bk_1\mu_0}{A}-\frac{B^2\mu_1}{A^2}-\frac{B\lambda\mu_0}{A}+\frac{2E\mu_0}{A}-\frac{2CE\mu_1}{A^2}-\frac{2Bk_0\mu_1}{A}-\frac{4Ek_1\mu_1}{A}-\frac{6BE\mu_2}{A^2}-\frac{2\lambda E\mu_2}{A}-\frac{4Ek_0\mu_2}{A}=0 \tag{14}$$

$$2k_1\mu_0-\frac{2Ck_1\mu_0}{A}+\frac{3B\mu_1}{A}+\lambda\mu_1-\frac{3B\lambda\mu_1}{A^2}-\frac{\lambda C\mu_1}{A}+2k_0\mu_1-\frac{2\lambda k_0\mu_1}{A}-\frac{4Bk_1\mu_1}{A}- \\ \frac{4B^2\mu_2}{A^2}-\frac{2B\lambda\mu_2}{A}+\frac{8E\mu_2}{A}-\frac{8\lambda E\mu_2}{A^2}-\frac{4Bk_0\mu_2}{A}-\frac{6Ek_1\mu_2}{A}=0 \tag{15}$$

$$-2\mu_1+\frac{4C\mu_1}{A}-\frac{2C^2\mu_1}{A^2}+4k_1\mu_1-\frac{4Ck_1\mu_1}{A}+\frac{10B\mu_2}{A}+2\lambda\mu_2-\frac{10BC\mu_2}{A^2}- \\ \frac{2\lambda C\mu_2}{A}+4k_0\mu_2-\frac{4Ck_0\mu_2}{A}-\frac{6Bk_1\mu_2}{A}=0 \tag{16}$$

$$-6\mu_2+\frac{12C\mu_2}{A}-\frac{6C^2\mu_2}{A^2}+6k_1\mu_2-\frac{6Ck_1\mu_2}{A}=0 \tag{17}$$

Solving the last system of equations we get

$$k_0=\frac{-B-Ac}{2A},k_1=\frac{A-C}{A},\mu_2=-k_1^2,\mu_0=-\frac{ek_1^3}{-A+C+Ck_1},\mu_2=-ck_1-2k_0k_1 \tag{18}$$

Thus we obtain the following solutions of Eq. (12):

For $B\neq 0, \Psi=A-C$ and $\Omega=B^2+4E(A-C)>0$, we get

$$u_1(\zeta)=\frac{B+2Ak_0\sqrt{\Omega}\text{Tanh}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]}{2A}$$
$$v_1(\zeta)=-\frac{B^2+2AB(c+2k_0)-4A^2\mu_0+2(B+A(c+2k_0))\sqrt{\Omega}\text{Tanh}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]+\Omega\text{Tanh}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]^2}{4A^2} \tag{19}$$

and

$$u_2(\zeta) = \frac{B + 2Ak_0\sqrt{\Omega}\mathrm{Coth}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]}{2A}$$
$$v_2(\zeta) = -\frac{B^2 + 2AB(c+2k_0) - 4A^2\mu_0 + 2(B + A(c+2k_0))\sqrt{\Omega}\mathrm{Coth}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right] + \Omega\mathrm{Coth}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]^2}{4A^2} \tag{20}$$

However for $B \neq 0, \Psi = A - C$ and $\Omega = B^2 + 4E(A - C) < 0$, we obtain periodic solutions

$$u_3(\zeta) = \frac{B + 2Ak_0\sqrt{-\Omega}\mathrm{Tan}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]}{2A}$$
$$v_3(\zeta) = -\frac{B^2 + 2AB(c+2k_0) - 4A^2\mu_0 + 2(B + A(c+2k_0))\sqrt{-\Omega}\mathrm{Tan}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right] - \Omega\mathrm{Tan}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]^2}{4A^2} \tag{21}$$

$$u_4(\zeta) = \frac{B + 2Ak_0\sqrt{-\Omega}\mathrm{Cot}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]}{2A}$$
$$v_4(\zeta) = -\frac{B^2 + 2AB(c+2k_0) - 4A^2\mu_0 + 2(B + A(c+2k_0))\sqrt{-\Omega}\mathrm{Cot}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right] - \Omega\mathrm{Cot}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]^2}{4A^2} \tag{22}$$

For $B \neq 0, \Psi = A - C$ and $\Omega = B^2 + 4E(A - C) = 0$, we obtain rational solutions

$$u_5(\zeta) = k_0 + \frac{\frac{B}{2} + \frac{\psi}{\zeta}}{A}$$
$$v_5(\zeta) = \mu_0 - \frac{(c+2k_0)(B\zeta + 2\psi)}{2A\zeta} - \frac{(B\zeta + 2\psi)^2}{4A^2\zeta^2} \tag{23}$$

For $B = 0, \Psi = A - C$ and $\Delta = \Psi E > 0$, we get

$$u_6(\zeta) = \frac{\lambda}{2} + \frac{\sqrt{\Delta}\text{Tanh}\left[\frac{E\zeta}{\sqrt{\Delta}}\right]}{A}$$

$$v_6(\zeta) = \frac{\Delta\left[1 - \text{Tanh}\left[\frac{E\zeta}{\sqrt{\Delta}}\right]^2\right]}{A^2} \tag{24}$$

$$u_7(\zeta) = \frac{\lambda}{2} + \frac{\sqrt{\Delta}\text{Coth}\left[\frac{E\zeta}{\sqrt{\Delta}}\right]}{A}$$

$$v_7(\zeta) = \frac{-\Delta\left[-1 + \text{Coth}\left[\frac{E\zeta}{\sqrt{\Delta}}\right]^2\right]}{A^2} \tag{25}$$

For $B = 0, \Psi = A - C$ and $\Delta = \Psi E < 0$, the periodic solutions

$$u_8(\zeta) = \frac{-\lambda}{2} - \frac{\sqrt{-\Delta}\text{Tan}\left[\frac{E\zeta}{\sqrt{-\Delta}}\right]}{A}$$

$$v_8(\zeta) = \frac{\Delta\left[-1 + \text{Tan}\left[\frac{E\zeta}{\sqrt{-\Delta}}\right]^2\right]}{A^2} \tag{26}$$

$$u_9(\zeta) = \frac{-\lambda}{2} - \frac{\sqrt{\Delta}\text{Cot}\left[\frac{E\zeta}{\sqrt{-\Delta}}\right]}{A}$$

$$v_9(\zeta) = \frac{\Delta\left[-1 + \text{Cot}\left[\frac{E\zeta}{\sqrt{-\Delta}}\right]^2\right]}{A^2} \tag{27}$$

2. The (2 + 1)-dimensional breaking soliton system

We consider the (2 + 1)-dimensional breaking soliton system,

$$u_t + \alpha u_{xxy} + 4\alpha(uv)_x = 0,$$

$$u_y = v_x \tag{28}$$

Balancing the highest derivative term with non-linear terms, hence we may assume that

$$u(x,y,t) = k_0 + k_1\phi(\zeta) + k_2\phi^2(\zeta)$$
$$v(x,y,t) = \mu_0 + \mu_1\phi(\zeta) + \mu_2\phi^2(\zeta) \tag{29}$$

where $\zeta = x + y - \lambda t$. By the same way we Substitute Eq. (29) into Eq. (28) and collecting coefficients of polynomial of ϕ^i and equating them to zero, we get a system of algebraic equations for k_0, k_1, k_2, μ_0, μ_1 and μ_2, by solving this system of equations we get the first set,

$$k_1 = \frac{3B(A-C)}{2A^2}, k_2 = \frac{3(A-C)^2}{2A^2}, \mu_1 = k_1, \mu_2 = k_2 \tag{30}$$

and the second set,

$$k_1 = 0, k2 = -\frac{3\left(A^2 - 2AC + C^2\right)}{2A^2}, \mu_2 = k_2, \mu_0 = \frac{-3Ak_0 + 3Ck_0 - 4ek_2}{3(A-C)}, \mu_1 = 0 \tag{31}$$

Thus for the first set, we obtain the follwing solutions of Eq. (6):

For $B \neq 0, \Psi = A - C$ and $\Omega = B^2 + 4E(A-C) > 0$, we get

$$u_1(\zeta) = k_0 + \frac{3\Omega \text{Tanh}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]^2}{8A^2}$$
$$v_1(\zeta) = \mu_0 + \frac{3\Omega \text{Tanh}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]^2}{8A^2} \tag{32}$$

and

$$u_2(\zeta) = k_0 + \frac{3\Omega \text{Coth}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]^2}{8A^2}$$
$$v_2(\zeta) = \mu_0 + \frac{3\Omega \text{Coth}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]^2}{8A^2} \tag{33}$$

However for $B \neq 0, \Psi = A - C$ and $\Omega = B^2 + 4E(A - C) < 0$, we obtain periodic solutions

$$
\begin{aligned}
u_3(\zeta) &= k_0 - \frac{3\Omega \mathrm{Tan}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]^2}{8A^2} \\
v_3(\zeta) &= \mu_0 - \frac{3\Omega \mathrm{Tan}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]^2}{8A^2}
\end{aligned}
\tag{34}
$$

$$
\begin{aligned}
u_4(\zeta) &= k_0 - \frac{3\Omega \mathrm{Cot}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]^2}{8A^2} \\
v_4(\zeta) &= \mu_0 - \frac{3\Omega \mathrm{Cot}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]^2}{8A^2}
\end{aligned}
\tag{35}
$$

For $B \neq 0, \Psi = A - C$ and $\Omega = B^2 + 4E(A - C) = 0$, we obtain rational solutions

$$
\begin{aligned}
u_5(\zeta) &= k_0 + \frac{3\left(3B^2\zeta^2 + 8B\zeta\psi + 4\psi^2\right)}{8A^2\zeta^2} \\
v_5(\zeta) &= \mu_0 + \frac{3\left(3B^2\zeta^2 + 8B\zeta\psi + 4\psi^2\right)}{8A^2\zeta^2}
\end{aligned}
\tag{36}
$$

For $B = 0, \Psi = A - C$ and $\Delta = \Psi E > 0$, we get

$$
\begin{aligned}
u_6(\zeta) &= k_0 + \frac{3\Delta \mathrm{Tanh}\left[\frac{E\zeta}{\sqrt{\Delta}}\right]^2}{2A^2} \\
v_6(\zeta) &= \mu_0 + \frac{3\Delta \mathrm{Tanh}\left[\frac{E\zeta}{\sqrt{\Delta}}\right]^2}{2A^2}
\end{aligned}
\tag{37}
$$

$$
\begin{aligned}
u_7(\zeta) &= k_0 + \frac{3\Delta \mathrm{Coth}\left[\frac{E\zeta}{\sqrt{\Delta}}\right]^2}{2A^2} \\
v_7(\zeta) &= \mu_0 + \frac{3\Delta \mathrm{Coth}\left[\frac{E\zeta}{\sqrt{\Delta}}\right]^2}{2A^2}
\end{aligned}
\tag{38}
$$

For $B=0, \Psi = A-C$ and $\Delta = \Psi E < 0$, the periodic solutions

$$u_8(\zeta) = k_0 - \frac{3\Delta \mathrm{Tan}\left[\frac{\sqrt{-\Delta}\zeta}{\Psi}\right]^2}{2A^2}$$
$$v_8(\zeta) = \mu_0 - \frac{3\Delta \mathrm{Tan}\left[\frac{\sqrt{-\Delta}\zeta}{\Psi}\right]^2}{2A^2} \tag{39}$$

$$u_9(\zeta) = k_0 - \frac{3\Delta \mathrm{Cot}\left[\frac{\sqrt{-\Delta}\zeta}{\Psi}\right]^2}{2A^2}$$
$$v_9(\zeta) = \mu_0 - \frac{3\Delta \mathrm{Cot}\left[\frac{\sqrt{-\Delta}\zeta}{\Psi}\right]^2}{2A^2} \tag{40}$$

For the second set,

For $B \neq 0, \Psi = A-C$ and $\Omega = B^2 + 4E(A-C) > 0$, we get

$$u_1(\zeta) = k_0 - \frac{3\Omega \mathrm{Tanh}\left[\frac{\zeta\sqrt{\Omega}}{2\Psi}\right]^2}{8A^2}$$
$$v_1(\zeta) = -k_0 + \frac{2\Delta}{A^2} - \frac{3\Omega \mathrm{Tanh}\left[\frac{\zeta\sqrt{\Omega}}{2\Psi}\right]^2}{8A^2} \tag{41}$$

and

$$u_2(\zeta) = k_0 - \frac{3\Omega \mathrm{Coth}\left[\frac{\zeta\sqrt{\Omega}}{2\Psi}\right]^2}{8A^2}$$
$$v_2(\zeta) = -k_0 + \frac{2\Delta}{A^2} - \frac{3\Omega \mathrm{Coth}\left[\frac{\zeta\sqrt{\Omega}}{2\Psi}\right]^2}{8A^2} \tag{42}$$

However for $B \neq 0, \Psi = A-C$ and $\Omega = B^2 + 4E(A-C) < 0$, we obtain periodic solutions

$$u_3(\zeta) = k_0 + \frac{3\Omega \mathrm{Tan}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]^2}{8A^2}$$
$$v_3(\zeta) = -k_0 + \frac{2e\psi}{A^2} + \frac{3\Omega \mathrm{Tan}\left[\frac{\zeta\sqrt{\Omega}}{2\psi}\right]^2}{8A^2} \tag{43}$$

$$u_4(\zeta) = k_0 + \frac{3\Omega \mathrm{Cot}\left[\frac{\zeta\sqrt{\Omega}}{2\Psi}\right]^2}{8A^2}$$

$$v_4(\zeta) = -k_0 + \frac{2\Delta}{A^2} + \frac{3\Omega \mathrm{Cot}\left[\frac{\zeta\sqrt{\Omega}}{2\Psi}\right]^2}{8A^2} \tag{44}$$

For $B \neq 0, \Psi = A - C$ and $\Omega = B^2 + 4E(A - C) = 0$, we obtain rational solutions

$$u_5(\zeta) = k_0$$

$$v_5(\zeta) = \frac{2E}{A} - \frac{2CE}{A^2} - k_0 \tag{45}$$

For $B = 0, \Psi = A - C$ and $\Delta = \Psi E > 0$, we get

$$u_6(\zeta) = k_0 - \frac{3\Delta \mathrm{Tanh}\left[\frac{E\zeta}{\sqrt{\Delta}}\right]^2}{2A^2}$$

$$v_6(\zeta) = -\frac{-4\Delta + 2A^2\mathrm{k0} + 3\Delta \mathrm{Tanh}\left[\frac{E\zeta}{\sqrt{\Delta}}\right]^2}{2A^2} \tag{46}$$

$$u_7(\zeta) = k_0 - \frac{3\Delta \mathrm{Coth}\left[\frac{E\zeta}{\sqrt{\Delta}}\right]^2}{2A^2}$$

$$v_7(\zeta) = -\frac{-4\Delta + 2A^2\mathrm{k0} + 3\Delta \mathrm{Coth}\left[\frac{E\zeta}{\sqrt{\Delta}}\right]^2}{2A^2} \tag{47}$$

For $B = 0, \Psi = A - C$ and $\Delta = \Psi E < 0$, the periodic solutions

$$u_8(\zeta) = k_0 + \frac{3\Delta \mathrm{Tan}\left[\frac{\sqrt{-\Delta}\zeta}{\Psi}\right]^2}{2A^2}$$

$$v_8(\zeta) = -\frac{-4\Delta + 2A^2\mathrm{k0} + 3\Delta \mathrm{Tan}\left[\frac{\sqrt{-\Delta}\zeta}{\Psi}\right]^2}{2A^2} \tag{48}$$

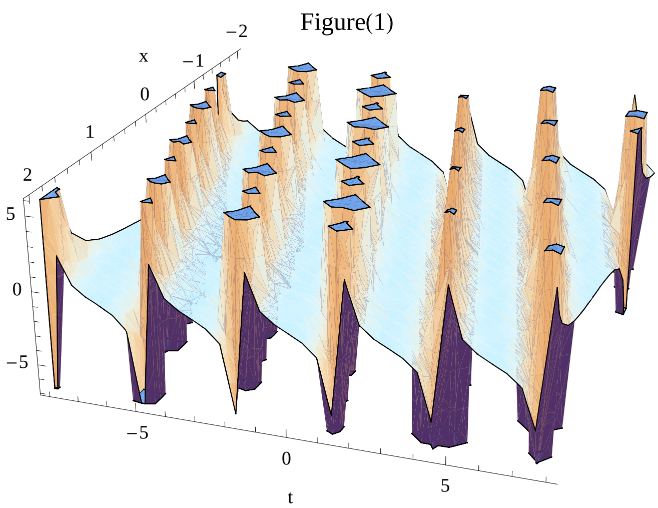

FIG. 1: Three-dimensional profile of the periodic solution

$$u_9(\zeta) = k_0 + \frac{3\Delta\text{Cot}\left[\frac{\sqrt{-\Delta}\zeta}{\Psi}\right]^2}{2A^2}$$
$$v_9(\zeta) = -\frac{-4\Delta + 2A^2\text{k0} + 3\Delta\text{Cot}\left[\frac{\sqrt{-\Delta}\zeta}{\Psi}\right]^2}{2A^2} \tag{49}$$

4. CONCLUSION

In this article, (G'/G)-expansion method was applied to give the traveling wave solutions of two Coupled $(2+1)$-Dimensional Equations, the BoitiLeonPempinelli (BLP) equation and the $(2+1)$-dimensional breaking soliton equation, The (G'/G)-expansion method examined for investigating the rogue wave solutions for (2+1) dimensional real field NLEEs.

(G'/G)-expansion method gives different classes of solutions. These solutions include many types like rational, periodical, soliton solutions, etc. For example, solutions (26) and (34) are examples exhibiting the sinusoidal-type periodical solutions, which develop a singularity at a finite point, i.e., for any fixed $t = t_0$ there exists a value of 0 at which these solutions blow up (see figure 1). Solution u_7 in (25) is in the form of explosive/blow-up solutions as depicted in figure 2.

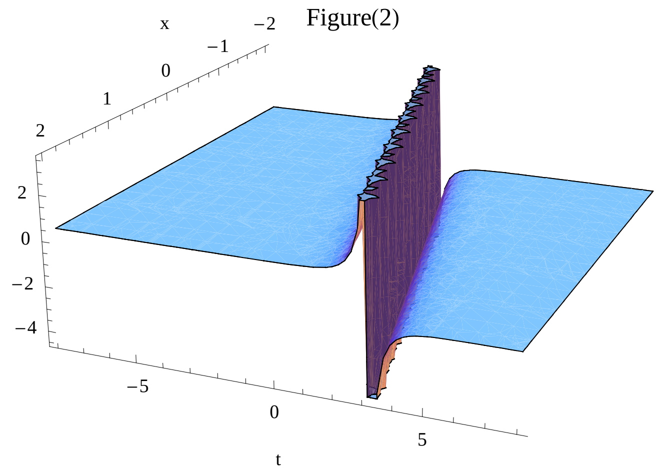

FIG. 2: Three-dimensional profile of the explosive/blowup pulse

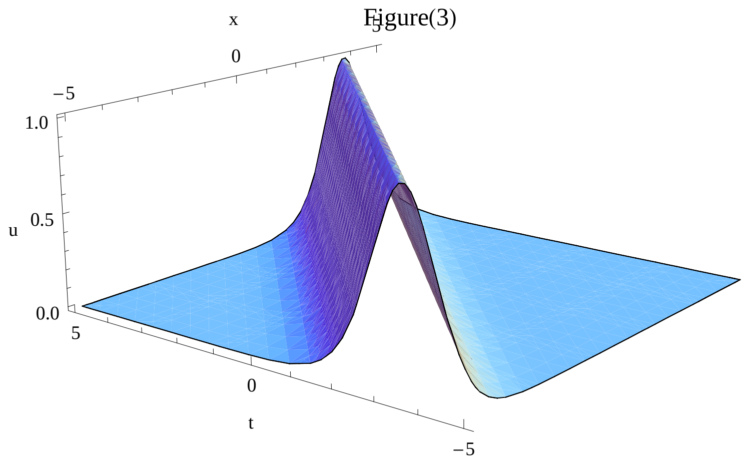

FIG. 3: Three-dimensional profile of the soliton solution.

Solution (23) represents the rational-type solutions, the rational solution may be a discrete joint union of manifolds. The solutions v_6 in (24) is a soliton wave solution (see figure 3), while we obtained shock wave solutions like u_6 in (24) as depicted in figure 4.

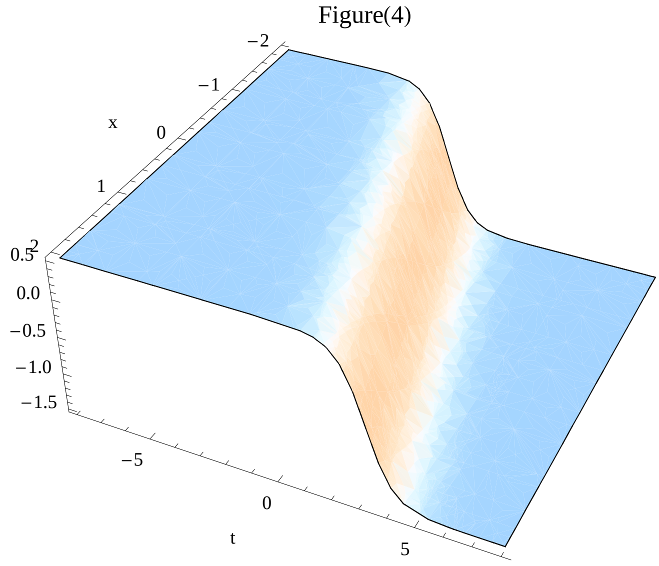

FIG. 4: Three-dimensional profile of the shock wave solution.

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