Entanglement of thermal states of two-qutrit states with Long-Range Interaction

A. R. Mohammed[1*] and T. M. El-Shahat[1†]
[1]Math. Dept., Faculty of Science, Al-Azhar University, 71524, Assiut, Egypt.

Abstract

We investigate the effect of the long-range interaction (LRI) with an inverse-square function on the thermal entanglement in anisotropic two-qutrit Heisenberg XYZ system with Dzyaloshinskii-Moriya (DM) interaction in the presence of the external magnetic field, using Negativity and Measurement-Induced Disturbance (MID) to quantify entanglement. The temperature and magnetic field dependence of the thermal entanglement in this system for this interaction are discussed. Our results indicate that, when the LRI type interactions exist, there is a rich conduct dependent between spins on the interaction strength, temperature, DM interaction and magnetic field.
In addition, we conclude that sudden death is displayed at the critical distance of the entanglement. We find that for less than a critical distance there are entanglement plateaus dependent upon the distance between spins. Furthermore it, we will make obvious comparison between the measurement-induced disturbance (MID) and negativity for this model. We will discover that MID is more robust than thermal entanglement against temperature T.

1 Introduction

The entanglement is a key asset which recognizes quantum information theory from the classical one. It assumes a focal part in numerous potential applications such as quantum computation, quantum information, quantum teleportation, quantum cryptography and dense coding [1]. In case of entangled subsystems, the subsystem states prevent isolation of the perfect state vector. This is the reason these subsystems are not any more autonomous, regardless of the possibility that they are far isolated spatially. An estimation on one subsystem not just gives information about the other subsystem, yet in addition, gives a probability of controlling it. Hence entanglement turns into the essential agent in quantum cryptography, quantum computations and information processing and teleportation, and so forth [2].
The describe two qubit interaction is least difficult by the Ising one between spin 1/2 particles as $J\sigma_1^z\sigma_2^z$. By the Heisenberg magnetic spin interaction models, the more broad interaction between two qubits is given. Temporarily in dense matter systems [3], these models have been widely tested amid quite a few years and hypothetically as precisely resolvable many bodies problems (Bethe, Baxter, and others) [4].
By generating entangled qubits and building quantum gates [5], presently they end up plainly encouraging to acknowledge information processing and quantum computation in a more broad setting than the magnetic chains.
Recently entanglement of two qubits [6] and its reliance on outer magnetic Fields, anisotropy, and temperature have been considered in a few Heisenberg models: the Ising model [7]; the XX and XY models [8], [9]; the XXX model [10]; the XXZ show [11]; and the XYZ model [12]. The Heisenberg XXX model, in case of the ferromagnetic $J < 0$ the spin states are unentangled, while they are entangled in case of antiferromagnetic $J > 0$ at adequately little temperature $T < T_c = \frac{2J}{kln3}$. The critical point in the investigation of such models is the means by which to build entanglement in

*E-mail a.radwan@azhar.edu.eg
†E-mail el_shahat@yahoo.com

a circumstance when it as of now exists or doesn't exist. The anisotropic antisymmetric exchange interaction is presented in phenomenological contentions by Dzialoshinskii [13] and in microscopic grounds by Moriya [14]. This interaction is expressed by $\vec{D}.[\vec{S_1} \times \vec{S_2}]$. A lot of authors introduced detailed and comprehensive studies have shown the dependence of entanglement on various parameters, for example, interaction strength, temperature, single-ion anisotropy and magnetic field [15]−[17]. The entanglement depends on the distance between spins, as well other physical parameters in case realistic systems. Such as short-range entanglement between spins or charge degrees of freedom. It observed in quantum dots, nanotubes or molecules.
Clearly, Due to lattice phonons at finite temperature, the positions of spins could oscillate in case realistic spin lattices. For this situation the integrals are exchange as a function of position and thus depend on the distance between spins. This reliance of the exchange interaction J(R) on the distance between spins may thus play an important role on the entanglement of the system. Such interaction types are known as Calogero-Moser type interactions, obtained in Haldane-Shastry model [18]. Understanding the detailed effect of Calogero-Moser type interactions on spin. Entanglement can, therefore, contribute the realistic assessment of the potential of such spin systems for solid state quantum computation and communication.
It found studies in the Heisenberg spin systems, shown that long-distance entanglement can be obtained using this interaction type for different values of the interaction parameter α, $J(R) \sim R^{-\alpha}$. It's known Qutrit systems attract much attention from researchers as they are promising candidates for quantum information processing (QIP) [19].
The paper is organised as follows. In Section 2 we descibe an anisotropic two-qutrit Heisenberg XYZ system, and give the eigenvalues, eigenvectors and density matrix of the system and discuss the numerical results of modelling, giving the Negativity and MID of the system for this interactions with differing model parameters. Finally, in section 3 we present our conclusions.

2 Model and solution for a two-qutrit

The Hamiltonian H for a two-qutrit anisotropic Heisenberg model with z-component interaction parameter D_z is

$$\begin{aligned} H &= J(1+\gamma)\sigma_1^x\sigma_2^x + J(1-\gamma)\sigma_1^y\sigma_2^y + J_z\sigma_1^z\sigma_2^z + D_z(\sigma_1^x\sigma_2^y - \sigma_1^y\sigma_2^x) \\ &+ B(\sigma_1^z + \sigma_2^z) + J(R)(\sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y) \end{aligned} \tag{1}$$

Where J and J_z are the real coupling parameter, γ is the anisotropic parameter. D_z is the z-component DM interaction parameter, and $\sigma^i (i = x, y, z)$ are Pauli matrices. B is the homogeneous part of the magnetic field. The DM interaction and external magnetic fields are thought to be along the z-direction. J(R) is the spin-spin coupling constant which will be defined in terms of the distance between spins as inverse-square function $J(R) = \frac{1}{R^2}$. All the parameters are dimensionless. We consider first the long-range interaction $G = J(R) = 1/R^2$ which is a version of the Haldane-Shastry model with exchange interaction.

The eigenvalues for this type of the long-range interaction are given by

$$
\begin{aligned}
E_1 &= -4Jz \\
E_{2,3} &= \mp 2\alpha_1 \\
E_{2,3} &= \mp 2\alpha_2 \\
E_{6,7} &= Jz - \frac{1}{2}\sqrt{\mu_1 \mp \frac{1}{2}\sqrt{\mu_2}} \\
E_{8,9} &= Jz + \frac{1}{2}\sqrt{\mu_3 \mp \frac{1}{2}\sqrt{\mu_4}} \\
|\psi_1\rangle &= \frac{1}{\mathrm{n1}}(A3|02\rangle + |20\rangle) \\
|\psi_2\rangle &= \frac{1}{\mathrm{n2}}(b2|01\rangle + b4|10\rangle + b6|12\rangle + |21\rangle) \\
|\psi_3\rangle &= \frac{1}{\mathrm{n3}}(c2|01\rangle + c4|10\rangle + c6|12\rangle + |21\rangle) \\
|\psi_4\rangle &= \frac{1}{\mathrm{n4}}(d2|01\rangle + d4|10\rangle + d6|12\rangle + |21\rangle) \\
|\psi_5\rangle &= \frac{1}{\mathrm{n5}}(e2|01\rangle + e4|10\rangle + e6|12\rangle + |21\rangle)
\end{aligned}
\tag{2}
$$

where

$$
\begin{aligned}
A1 &= \frac{\mathrm{Dz}^2 - \left(J + \frac{1}{R^2}\right)^2}{\mathrm{Dz}^2 + \left(J + \frac{1}{R^2}\right)^2} & A2 &= \frac{\mathrm{Dz}^2 - \left(J + \frac{1}{R^2}\right)^2}{\mathrm{Dz}^2 + \left(J + \frac{1}{R^2}\right)^2} \\
A3 &= \mathrm{A3} = \mathrm{A1} - i\mathrm{A2} & n1 &= \sqrt{\mathrm{A3}A3^* + 1} \\
b4 &= -\frac{-2\alpha 3 + B^2 - \alpha 1 B + 4\gamma^2 J^2}{2\alpha 1\gamma J} & B1 &= B\left(-\alpha 3 + 2\mathrm{Dz}^2 + 2(G+J)^2\right) \\
B2 &= \alpha 1\alpha 3 & B3 &= 2\alpha 1\gamma J(G+J) \\
B4 &= 2\alpha 1\gamma \mathrm{Dz}J & & \\
B5 &= -\frac{\mathrm{B3}(\mathrm{B1}+\mathrm{B2})}{\mathrm{B3}^2 + \mathrm{B4}^2} & B6 &= \frac{\mathrm{B4}(\mathrm{B1}+\mathrm{B2})}{\mathrm{B3}^2 + \mathrm{B4}^2} \\
b2 &= \mathrm{B5} - i\mathrm{B6} & B7 &= -\left(\alpha 3 - 2\mathrm{Dz}^2 - 2(G+J)^2\right) \\
B8 &= \alpha 1(-(G+J)) & B9 &= \alpha 1\mathrm{Dz} \\
B10 &= \frac{\mathrm{B7B8}}{\mathrm{B8}^2 + \mathrm{B9}^2} & B11 &= \frac{\mathrm{B7B9}}{\mathrm{B8}^2 + \mathrm{B9}^2} \\
b6 &= \mathrm{B10} - i\mathrm{B11} & n2 &= \sqrt{\mathrm{b2}b2^* + \mathrm{b4}^2 + \mathrm{b6}b6^* + 1} \\
C5 &= -\frac{\mathrm{B3}(\mathrm{B2}-\mathrm{B1})}{\mathrm{B3}^2 + \mathrm{B4}^2} & C6 &= \frac{\mathrm{B4}(\mathrm{B2}-\mathrm{B1})}{\mathrm{B3}^2 + \mathrm{B4}^2} \\
c2 &= \mathrm{C5} - i\mathrm{C6} & c6 &= -b6 \\
n3 &= \sqrt{\mathrm{b4}^2 + \mathrm{c2}c2^* + \mathrm{c6}c6^* + 1} & d4 &= -\frac{2\alpha 3 + B^2 - \alpha 2 B + 4\gamma^2 J^2}{2\alpha 2\gamma J} \\
D1 &= -\left(\alpha 3 + 2\mathrm{Dz}^2 + 2(G+J)^2\right) & D2 &= \alpha 2\alpha 3 \\
D3 &= 2\alpha 2\gamma J(G+J) & D4 &= 2\alpha 2\gamma \mathrm{Dz}J
\end{aligned}
$$

$$D5=\frac{\mathrm{D3}(\mathrm{D1}+\mathrm{D2})}{\mathrm{D3}^2+\mathrm{D4}^2} \qquad D6=\frac{\mathrm{D4}(\mathrm{D1}+\mathrm{D2})}{\mathrm{D3}^2+\mathrm{D4}^2}$$
$$d2=\mathrm{D5}+i\mathrm{D6} \qquad D7=\alpha 3+2\mathrm{Dz}^2+2(G+J)^2$$
$$D8=\alpha 2(-(G+J)) \qquad D9=\alpha 2\mathrm{Dz}$$
$$D10=\frac{\mathrm{D7D8}}{\mathrm{D8}^2+\mathrm{D9}^2} \qquad D11=\frac{\mathrm{D7D9}}{\mathrm{D8}^2+\mathrm{D9}^2}$$
$$d6=\mathrm{D10}-i\mathrm{D11} \qquad n4=\sqrt{\mathrm{d2}d2^*+\mathrm{d4}^2+\mathrm{d6}d6^*+1}$$
$$e4=-d4 \qquad E5=\frac{\mathrm{D3}\left(\mathrm{D2}-\frac{\mathrm{D1}}{B}\right)}{\mathrm{D3}^2+\mathrm{D4}^2}$$
$$E6=\frac{\mathrm{D4}\left(\mathrm{D2}-\frac{\mathrm{D1}}{B}\right)}{\mathrm{D3}^2+\mathrm{D4}^2} \qquad e2=\mathrm{E5}+i\mathrm{E6}$$
$$E8=\alpha 2(-(G+J)) \qquad E9=\alpha 2\mathrm{Dz}$$
$$E10=\frac{\mathrm{D1E8}}{\mathrm{E8}^2+\mathrm{E9}^2} \qquad E11=\frac{\mathrm{D1E9}}{\mathrm{E8}^2+\mathrm{E9}^2}$$
$$e6=\mathrm{E10}-i\mathrm{E11} \qquad n5=\sqrt{\mathrm{e2}e2^*+\mathrm{e4}^2+\mathrm{e6}e6^*+1}$$

$$\alpha_{1,2}=\sqrt{4\left(-\sqrt{(B^2+4\gamma^2J^2)\left(\mathrm{Dz}^2+(J+G)^2\right)}\mp\mathrm{Dz}^2+\gamma^2J^2+(J+G)^2\right)+B^2}$$
$$\alpha_3=\sqrt{(B^2+4\gamma^2J^2)\left(\mathrm{Dz}^2+(G+J)^2\right)}$$

the dynamics of the density operator

$$\rho(T)=\frac{1}{z}\sum_{i=1}^{9}(\exp(\frac{-E_i}{T}))|\psi_i\rangle\langle\psi_i|, \qquad where \quad z=Tr(\rho(T)) \tag{3}$$

the density matrix for the two-qutrit system is obtained as follows after straight calculations:

$$\rho(T)=\frac{1}{z}\begin{pmatrix} 0&0&0&0&0&0&0&0&0\\ 0&\rho_{22}&0&\rho_{24}&0&\rho_{26}&0&\rho_{28}&0\\ 0&0&\rho_{33}&0&0&0&\rho_{37}&0&0\\ 0&\rho_{42}&0&\rho_{44}&0&\rho_{46}&0&\rho_{48}&0\\ 0&0&0&0&0&0&0&0&0\\ 0&\rho_{62}&0&\rho_{64}&0&\rho_{66}&0&\rho_{68}&0\\ 0&0&\rho_{73}&0&0&0&\rho_{77}&0&0\\ 0&\rho_{82}&0&\rho_{84}&0&\rho_{86}&0&\rho_{88}&0\\ 0&0&0&0&0&0&0&0&0 \end{pmatrix} \tag{4}$$

has matrix elements given as follows:

$$\rho_{22}=\mathrm{q22}+\mathrm{r22}+\mathrm{u22}+\mathrm{w22} \qquad \rho_{24}=\mathrm{q24}+\mathrm{r24}+\mathrm{u24}+\mathrm{w24}$$
$$\rho_{26}=\mathrm{q26}+\mathrm{r26}+\mathrm{u26}+\mathrm{w26} \qquad \rho_{28}=\mathrm{q28}+\mathrm{r28}+\mathrm{u28}+\mathrm{w28}$$
$$\rho_{33}=\mathrm{t33} \qquad \rho_{37}=\mathrm{t37}$$
$$\rho_{42}=\mathrm{q42}+\mathrm{r42}+\mathrm{u42}+\mathrm{w42} \qquad \rho_{44}=\mathrm{q44}+\mathrm{r44}+\mathrm{u44}+\mathrm{w44}$$
$$\rho_{46}=\mathrm{q46}+\mathrm{r46}+\mathrm{u46}+\mathrm{w46} \qquad \rho_{48}=\mathrm{q48}+\mathrm{r48}+\mathrm{u48}+\mathrm{w48}$$
$$\rho_{62}=\mathrm{q62}+\mathrm{r62}+\mathrm{u62}+\mathrm{w62} \qquad \rho_{64}=\mathrm{q64}+\mathrm{r64}+\mathrm{u64}+\mathrm{w64}$$

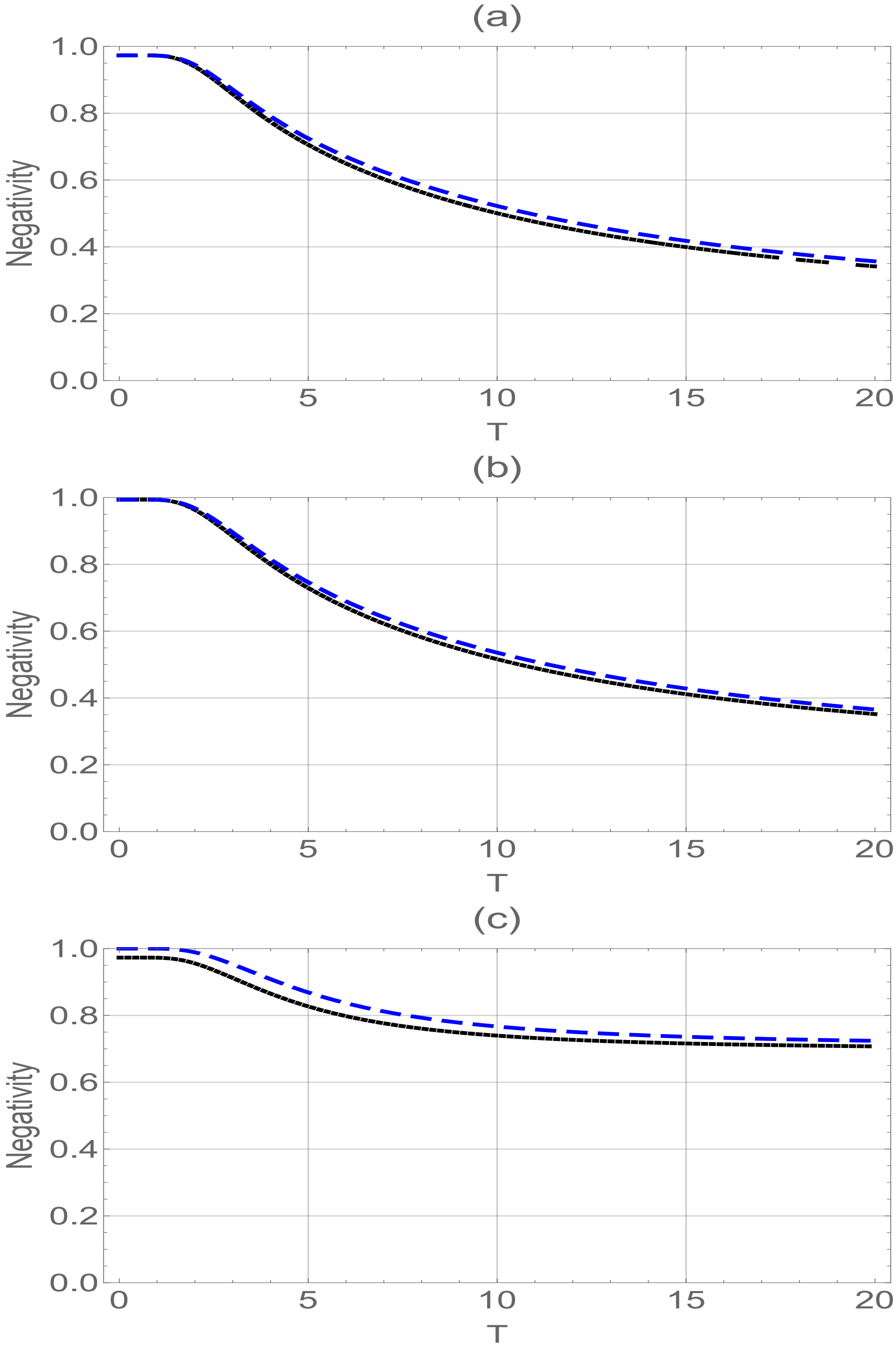


Figure 1: Negativity as a function of T in the case $J = 1/R^2$. (a) The solid and dashed curves are evaluated for $Dz = 0.5, 1$, respectively at $R = 1$ for $B = 0$. (b)The dotted and dashed curves are evaluated for $Dz = 0.5, 1$, respectively at $R = 1$ for $B = 1$. (c)The dashed, dotted and solid curves are evaluated for $B = 0, 0.5, 1$, respectively at $R = 0.01, Dz = 0.01$. ($\gamma = 1$,$J = 1$,$Jz = 1$)

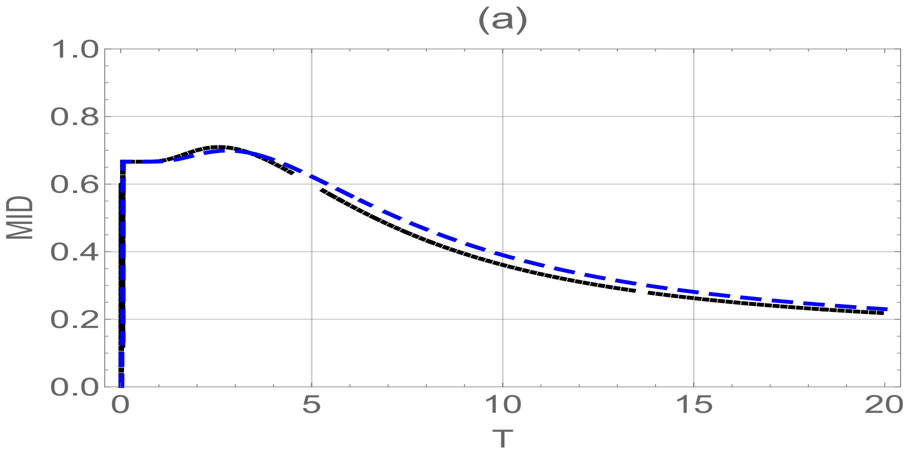
$$\rho_{66} = \mathrm{q66} + \mathrm{r66} + \mathrm{u66} + \mathrm{w66} \qquad \rho_{68} = \mathrm{q68} + \mathrm{r68} + \mathrm{u68} + \mathrm{w68}$$

$$\rho_{73} = t73 \qquad \rho_{74} = t77$$

$$\rho_{82} = \mathrm{q82} + \mathrm{r82} + \mathrm{u82} + \mathrm{w82} \qquad \rho_{84} = \mathrm{q84} + \mathrm{r84} + \mathrm{u84} + \mathrm{w84}$$

$$\rho_{86} = \mathrm{q86} + \mathrm{r86} + \mathrm{u86} + \mathrm{w86} \qquad \rho_{88} = \mathrm{q88} + \mathrm{r88} + \mathrm{u88} + \mathrm{w88}$$

$$\begin{aligned} z = {} & \mathrm{q22} + \mathrm{q44} + \mathrm{q66} + \mathrm{q88} + \mathrm{r22} + \\ & \mathrm{r44} + \mathrm{r66} + \mathrm{r88} + \mathrm{t33} + \mathrm{t77} + \\ & \mathrm{u22} + \mathrm{u44} + \mathrm{u66} + \mathrm{u88} + \mathrm{w22} + \mathrm{w44} + \mathrm{w66} + \mathrm{w88} \end{aligned}$$



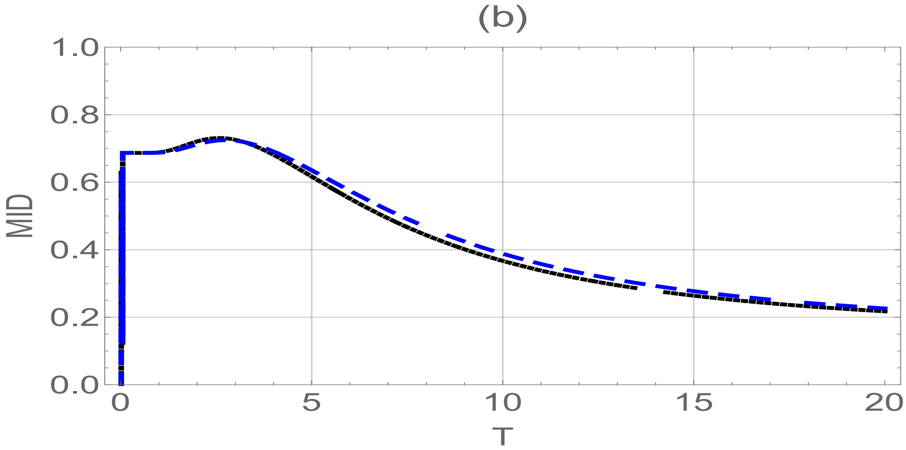


Figure 2: MID as a function of T in the case $J = 1/R^2$. The solid and dashed curves are evaluated for $Dz = 0.1, 1$, respectively at $R = 1$ for (a)$B = 0$ (b)$B = 0.5$. ($\gamma = 1$,$J = 1$,$Jz = 1$)

where

$$\mathrm{t33} = \frac{\mathrm{A3A4}e^{-\frac{\mathrm{E1}}{T}}}{\mathrm{n1}^2} \qquad \mathrm{t73} = \frac{\mathrm{A4}e^{-\frac{\mathrm{E1}}{T}}}{\mathrm{n1}^2} \qquad \mathrm{t37} = \frac{\mathrm{A3}e^{-\frac{\mathrm{E1}}{T}}}{\mathrm{n1}^2}$$

$$\mathrm{t77} = \frac{e^{-\frac{\mathrm{E1}}{T}}}{\mathrm{n1}^2} \qquad \mathrm{r22} = \frac{\mathrm{b2}b2^*e^{-\frac{\mathrm{E2}}{T}}}{\mathrm{n2}^2} \qquad \mathrm{r24} = \frac{\mathrm{b2b4}e^{-\frac{\mathrm{E2}}{T}}}{\mathrm{n2}^2}$$

$$\mathrm{r26} = \frac{\mathrm{b2}b6^*e^{-\frac{\mathrm{E2}}{T}}}{\mathrm{n2}^2} \qquad \mathrm{r28} = \frac{\mathrm{b2}e^{-\frac{\mathrm{E2}}{T}}}{\mathrm{n2}^2} \qquad \mathrm{r42} = \frac{\mathrm{b4}b2^*e^{-\frac{\mathrm{E2}}{T}}}{\mathrm{n2}^2}$$

$$\mathrm{r44} = \frac{\mathrm{b4}^2e^{-\frac{\mathrm{E2}}{T}}}{\mathrm{n2}^2} \qquad \mathrm{r46} = \frac{\mathrm{b4}b6^*e^{-\frac{\mathrm{E2}}{T}}}{\mathrm{n2}^2} \qquad \mathrm{r48} = \frac{\mathrm{b4}e^{-\frac{\mathrm{E2}}{T}}}{\mathrm{n2}^2}$$

$$\mathrm{r62} = \frac{\mathrm{b6}b2^*e^{-\frac{\mathrm{E2}}{T}}}{\mathrm{n2}^2} \qquad \mathrm{r64} = \frac{\mathrm{b4b6}e^{-\frac{\mathrm{E2}}{T}}}{\mathrm{n2}^2} \qquad \mathrm{r66} = \frac{\mathrm{b6}b6^*e^{-\frac{\mathrm{E2}}{T}}}{\mathrm{n2}^2}$$

$$\mathrm{r68} = \frac{\mathrm{b6}e^{-\frac{\mathrm{E2}}{T}}}{\mathrm{n2}^2} \qquad \mathrm{r82} = \frac{b2^*e^{-\frac{\mathrm{E2}}{T}}}{\mathrm{n2}^2} \qquad \mathrm{r84} = \frac{\mathrm{b4}e^{-\frac{\mathrm{E2}}{T}}}{\mathrm{n2}^2}$$

$$\mathrm{r86} = \frac{b6^*e^{-\frac{\mathrm{E2}}{T}}}{\mathrm{n2}^2} \qquad \mathrm{r88} = \frac{e^{-\frac{\mathrm{E2}}{T}}}{\mathrm{n2}^2} \qquad \mathrm{w22} = \frac{\mathrm{c2}c2^*e^{-\frac{\mathrm{E3}}{T}}}{\mathrm{n3}^2}$$

$$\mathrm{w24} = \frac{\mathrm{c2c4}e^{-\frac{\mathrm{E3}}{T}}}{\mathrm{n3}^2} \qquad \mathrm{w26} = \frac{\mathrm{c2}c6^*e^{-\frac{\mathrm{E3}}{T}}}{\mathrm{n3}^2} \qquad \mathrm{w28} = \frac{\mathrm{c2}e^{-\frac{\mathrm{E3}}{T}}}{\mathrm{n3}^2}$$

$$\mathrm{w42} = \frac{\mathrm{c4}c2^*e^{-\frac{\mathrm{E3}}{T}}}{\mathrm{n3}^2} \qquad \mathrm{w44} = \frac{\mathrm{c4}^2e^{-\frac{\mathrm{E3}}{T}}}{\mathrm{n3}^2} \qquad \mathrm{w46} = \frac{\mathrm{c4}c6^*e^{-\frac{\mathrm{E3}}{T}}}{\mathrm{n3}^2}$$

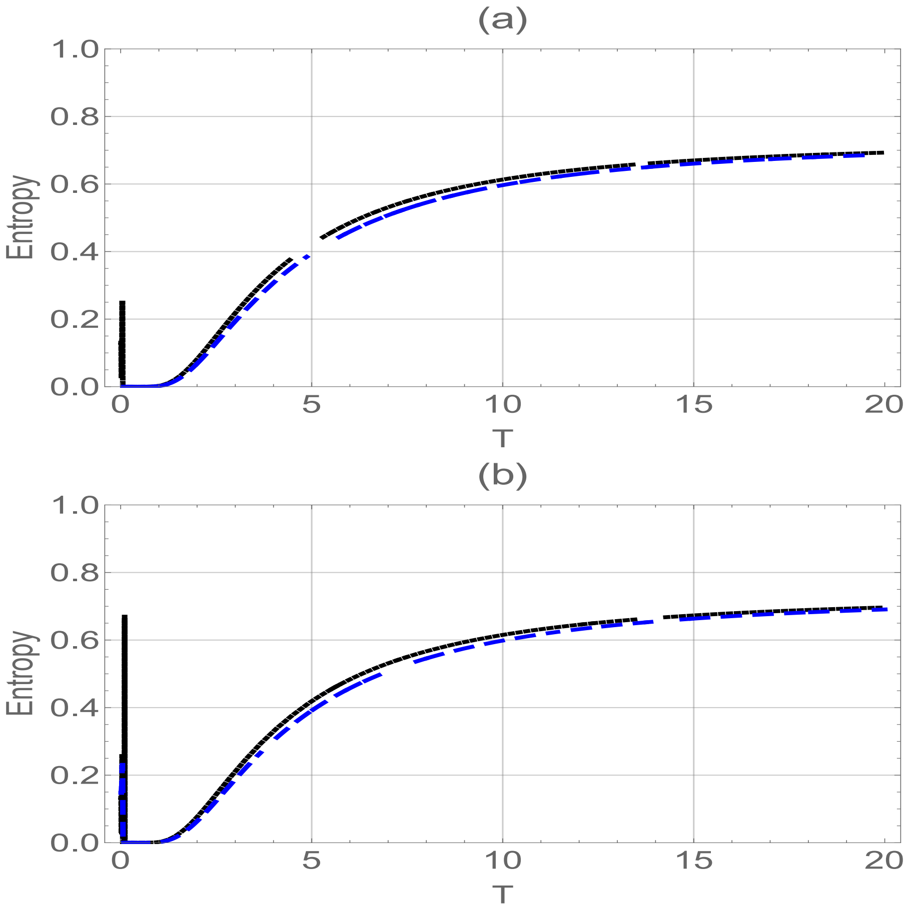


Figure 3: Entropy as a function of T in the case $J = 1/R^2$. The solid and dashed curves are evaluated for $Dz = 0.1, 1$, respectively at $R = 1$ for (a)$B = 0$ (b)$B = 0.5$. ($\gamma = 1$,$J = 1$,$Jz = 1$)

$$\mathrm{w48} = \frac{\mathrm{c4}e^{-\frac{\mathrm{E3}}{T}}}{\mathrm{n3}^2} \qquad \mathrm{w62} = \frac{\mathrm{c6}c2^*e^{-\frac{\mathrm{E3}}{T}}}{\mathrm{n3}^2} \qquad \mathrm{w64} = \frac{\mathrm{c4c6}e^{-\frac{\mathrm{E3}}{T}}}{\mathrm{n3}^2}$$

$$\mathrm{w66} = \frac{\mathrm{c6}c6^*e^{-\frac{\mathrm{E3}}{T}}}{\mathrm{n3}^2} \qquad \mathrm{w68} = \frac{\mathrm{c6}e^{-\frac{\mathrm{E3}}{T}}}{\mathrm{n3}^2} \qquad \mathrm{w82} = \frac{c2^*e^{-\frac{\mathrm{E3}}{T}}}{\mathrm{n3}^2}$$

$$\mathrm{w84} = \frac{\mathrm{c4}e^{-\frac{\mathrm{E3}}{T}}}{\mathrm{n3}^2} \qquad \mathrm{w86} = \frac{c6^*e^{-\frac{\mathrm{E3}}{T}}}{\mathrm{n3}^2} \qquad \mathrm{w88} = \frac{e^{-\frac{\mathrm{E3}}{T}}}{\mathrm{n3}^2}$$

$$\mathrm{q22} = \frac{\mathrm{d2}d2^*e^{-\frac{\mathrm{E4}}{T}}}{\mathrm{n4}^2} \qquad \mathrm{q24} = \frac{\mathrm{d2d4}e^{-\frac{\mathrm{E4}}{T}}}{\mathrm{n4}^2} \qquad \mathrm{q26} = \frac{\mathrm{d2}d6^*e^{-\frac{\mathrm{E4}}{T}}}{\mathrm{n4}^2}$$

$$\mathrm{q28} = \frac{\mathrm{d2}e^{-\frac{\mathrm{E4}}{T}}}{\mathrm{n4}^2} \qquad \mathrm{q42} = \frac{\mathrm{d4}d2^*e^{-\frac{\mathrm{E4}}{T}}}{\mathrm{n4}^2} \qquad \mathrm{q44} = \frac{\mathrm{d4}^2e^{-\frac{\mathrm{E4}}{T}}}{\mathrm{n4}^2}$$

$$\mathrm{q46} = \frac{\mathrm{d4}d6^*e^{-\frac{\mathrm{E4}}{T}}}{\mathrm{n4}^2} \qquad \mathrm{q48} = \frac{\mathrm{d4}e^{-\frac{\mathrm{E4}}{T}}}{\mathrm{n4}^2} \qquad \mathrm{q62} = \frac{\mathrm{d6}d2^*e^{-\frac{\mathrm{E4}}{T}}}{\mathrm{n4}^2}$$

$$\mathrm{q64} = \frac{\mathrm{d4d6}e^{-\frac{\mathrm{E4}}{T}}}{\mathrm{n4}^2} \qquad \mathrm{q66} = \frac{\mathrm{d6}d6^*e^{-\frac{\mathrm{E4}}{T}}}{\mathrm{n4}^2} \qquad \mathrm{q68} = \frac{\mathrm{d6}e^{-\frac{\mathrm{E4}}{T}}}{\mathrm{n4}^2}$$

$$\mathrm{q82} = \frac{d2^*e^{-\frac{\mathrm{E4}}{T}}}{\mathrm{n4}^2} \qquad \mathrm{q84} = \frac{\mathrm{d4}e^{-\frac{\mathrm{E4}}{T}}}{\mathrm{n4}^2} \qquad \mathrm{q86} = \frac{d6^*e^{-\frac{\mathrm{E4}}{T}}}{\mathrm{n4}^2}$$

$$\mathrm{q88} = \frac{e^{-\frac{\mathrm{E4}}{T}}}{\mathrm{n4}^2} \qquad \mathrm{u22} = \frac{\mathrm{e2}e2^*e^{-\frac{\mathrm{E5}}{T}}}{\mathrm{n5}^2} \qquad \mathrm{u24} = \frac{\mathrm{e2e4}e^{-\frac{\mathrm{E5}}{T}}}{\mathrm{n5}^2}$$

$$\mathrm{u26} = \frac{\mathrm{e2}e6^*e^{-\frac{\mathrm{E5}}{T}}}{\mathrm{n5}^2} \qquad \mathrm{u28} = \frac{\mathrm{e2}e^{-\frac{\mathrm{E5}}{T}}}{\mathrm{n5}^2} \qquad \mathrm{u42} = \frac{\mathrm{e4}e2^*e^{-\frac{\mathrm{E5}}{T}}}{\mathrm{n5}^2}$$

$$\mathrm{u44} = \frac{\mathrm{e4}^2e^{-\frac{\mathrm{E5}}{T}}}{\mathrm{n5}^2} \qquad \mathrm{u46} = \frac{\mathrm{e4}e6^*e^{-\frac{\mathrm{E5}}{T}}}{\mathrm{n5}^2} \qquad \mathrm{u48} = \frac{\mathrm{e4}e^{-\frac{\mathrm{E5}}{T}}}{\mathrm{n5}^2}$$

$$\mathrm{u62} = \frac{\mathrm{e6}e2^*e^{-\frac{\mathrm{E5}}{T}}}{\mathrm{n5}^2} \qquad \mathrm{u64} = \frac{\mathrm{e4e6}e^{-\frac{\mathrm{E5}}{T}}}{\mathrm{n5}^2} \qquad \mathrm{u66} = \frac{\mathrm{e6}e6^*e^{-\frac{\mathrm{E5}}{T}}}{\mathrm{n5}^2}$$

$$\mathrm{u68} = \frac{\mathrm{e6}e^{-\frac{\mathrm{E5}}{T}}}{} \qquad \mathrm{u82} = \frac{e2^*e^{-\frac{\mathrm{E5}}{T}}}{} \qquad \mathrm{u84} = \frac{\mathrm{e4}e^{-\frac{\mathrm{E5}}{T}}}{}$$

It is found that there are some systems could be entangled but the quantum correlations will be her zero Negativity and non-zero values. This means that Negativity use cannot be predicted as a measure of the entanglement of some quantum correlation. However, one can quantify the surprised quantum correlation by using the measurement-induced disturbance (MID) as a measure of quantum correlation [20, 21]. This measure expresses that if $\{\lambda_\mu\}$ refer to the eigenvalues of $\rho_{ab}^{T_2}$, then the negativity is given by, $N = \sum_{\mu=1}^{4} |\lambda_\mu| - 1$ where T_2 refers to the partial transposition for the second subsystem. By utilizing this definition for our system, the Negativity can be calculated explicitly as, $N = Max[0, -2Min[\lambda_i]]$. Thus in this letter, we use negativity as our measure of entanglement. the values of N range from zero to one: For a maximally-entangled when $N = 1$, while for a unentangled state $N = 0$[22].
Measurement-induced disturbance (MID) was defined as the difference between the quantum mutual information of a given quantum state and the classical state that is closest to the original quantum state. The quantum correlation can be quantified by the measurementinduced disturbance $Q(\rho) = I(\rho) - I(\prod(\rho))$. The total correlation in a bipartite state ρ can be well quantified by the quantum mutual information $I(\rho) = S(\rho^a) + S(\rho^b) - S(\rho)$, and $I(\prod(\rho))$, quantifies the classical correlations in ρ since $\prod(\rho)$ is a classical state. Where

$$\prod(\rho) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_{44} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{77} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{88} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{5}$$

. Here $S(\rho) = -\sum_i \lambda_i \log_2 \lambda_i$ denotes the von Neumann entropy. The von Neumann entropy, named after John von Neumann, is the extension of classical Gibbs entropy concepts to the field of quantum mechanics. It is one of the simplest entanglement measures. It vanishes for a pure state, where all populations are 0 or 1 and it reaches its maximum for the completely mixed state.

In Figure 1(a) The negativity of the two-qutrit system in the case $J = 1/R^2$ is plotted versus T for different DM interaction values at the absence of a magnetic field. We see that Negativity decrease with increase temperature. When increased DM interaction, we can see Negativity vanishes in an asymptotic way. The evolution of Negativity in terms of B for different values of D are plotted in Figure 1(b). It is obvious that, with the DM interaction Dz increasing, the range in which the quantum correlation exists become wider. In Figure 1(c) we studied the effect of the magnetic field on Negativity in the presence of both DM interaction and long-range interaction, we saw that negativity dropping suddenly with increasing T at a critical T_c value and with increased the B interaction, it is seen that decay faster before it reaches steady-going values. In Figure 2(a) The MID of the two-qutrit system in this interaction is plotted versus T for different DM interaction values at the non-attendance of a magnetic field. Reaches a plateau for small T at a fixed DM interaction and fixed long-range interaction. With increase DM interaction, MID Stretching and it disappears slowly. In the presence of a magnetic field with an increase, it found that MID expands more and vanishes in an asymptotic way as shown in Figure 2(b). These are the entanglement plateaus, i.e., the regions within the entanglement curve where the entanglement remains unchanged with increasing distance between spins. In Figure 3(a) we plot Entropy as a function of T, We noticed that when the magnetic field is missing, it suddenly dies and gives birth again. As the temperature increases the entanglement increases slowly until it reaches steady-going values. In the presence of the magnetic field the entanglement begin to value higher than the absence of the magnetic field and decrease with the increase of the DM interaction where the phenomenon of death and living at a lower temperature. With increasing temperature the entanglement increases slowly until it reaches constant-going values in Figure 3(b).

3 Conclusions

In this paper, the discussion centers on effect of the long-range interaction with an inverse-square on the thermal Negativity and MID in a two-qutrit via a Heisenberg XYZ model with different DM interaction under the external magnetic field. The numerical results show that, in the presence of the long-range interaction, thermal entanglement between spins has a Strong behavior dependent upon the magnetic field, DM interaction, temperature and interaction strength. We conclude that sudden death is displayed at the critical distance of the entanglement. Clearly the effect of long-range interaction on the resource of entanglement provides a rich source of behaviour, with maximum entanglement existing over significant parameter regimes with the associated entanglement sudden death. Given the major interest in spin systems for solid state quantum computation and communication, hybrid spin XYZ models could provide significant input to these applications. By using these parameters, entanglement can be controlled and changed. The long-range entanglement plays an important role in teleportation, quantum computing etc. We believe that these interesting results can be valuable for researcher in this area of physics. From the above, we conclude that MID may act as a more general tool than negativity for discussing quantum correlations quality of the systems.

References

[1] C. H. Bennet and D. P. DiVincenzo (2000) Nature, 404, pp. 247.

[2] D. G. Angelakis, M. Christandl, A. Ekert, A. Kay, and S. Kulik (2006) Volume 199, IOS Press.

[3] V. G. Baryakhtar, P. E. Wigen, and N. A. Lesnik (Eds.) (1998) Kluwer Academic Press.

[4] R. J. Baxter (1982) Academic Press.

[5] S.-B. Zheng, G.-C. Guo (2000) Phys. Rev. Lett., 85.

[6] Wootters, William K. Physical Review Letters 80.10 (1998): 2245.

[7] A. M. Childs, D.W. Leung, F. Verstraete, and G. Vidal (2003) Quantum Inf. Comput., 3, pp. 97.

[8] S. D. Hamieh and M. I. Katsnelson (2005) Phys. Rev. A, 72, 032316.

[9] Y. Sun, Y. Chen, and H. Chen (2003) Phys. Rev. A, 68.

[10] M. C., Arnesen, S. Bose and V. Vedral (2001) Phys. Rew. Lett., 87.

[11] X. Wang (2001) Phys. Lett. A, 281, pp.101.

[12] Zhou L., Song H. , Guo Y., Li C. (2003) Physical Review A, 68(2), 024301.

[13] I. Dzialoshinski (1958) Phys. Chem. Solids 4, pp.241.

[14] T. Moriya (1960) Phys. Rev. Lett. 120, pp.91.

[15] E. Albayrak, Eur. Phys. B 72, 491 (2009); Opt. Commun. 284, 1631 (2011)

[16] E. Ercolessi, S. Evangelisti, F. Ravanini, Phys. Lett. A 374, 2101 (2010).

[17] T. Chen, et al., Chin. Phys. B 19, 050302 (2010).

[18] B. S. Shastry, Phys. Rev. Lett. 60, 639 (1988).

[19] Zidan, N. (2016) International Journal of Theoretical Physics, 55(2), 1274-1284.

[20] Luo, S. Phys. Rev. A 77, 022301 (2008)

[21] Song, L., Yang, G.-H. Phys. 53, 1985 (2014).

[22] Mohammed, A.R. and El-Shahat, T.M. (2017) Journal of Quantum Information Science, 7, 160-171.