On nano pre-I-open sets and a decomposition of nano I-continuity

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Abstract

In this paper, we introduce and study the notion of nano pre-I-open sets which is properly placed between nano openness and nano preopenness regardless the nano topological ideal. Also, we show that the class of nano pre-I-open sets is properly places between the classes of nano I -open and nano preopen sets. We give a decomposition of nano I-continuity by proving that a function $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ is nano I-continuous if and only if it is nano pre-I-continuous and nano *-I-continuous.

Keywords and Phrases: Nano local functions, nano topological ideal, nano I-open set, nano pre-I-open set, nano I-continuous function, nano pre-I-continuous function.
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1 Introduction and preliminaries

Ideals in topological spaces have been considered since 1930 by kuratowski [1]. The topic has won its importance by the paper of Vaidyanathaswamy [2] in 1945. A nonempty collection of subsets of X with hereditary and finite additivity conditions is called an ideal or a dual filter on X. Namely a nonempty family $I \subseteq P(X)$ (where $P(X)$ is the set of all subsets of X) is called an ideal if and only if (i) $A \in I$ gives $P(A) \subseteq I$ and (ii) $A, B \in I$ gives $A \cup B \in I$. Given a topological space (X, τ) with an ideal I on X, a set operator $()^* : P(X) \to P(X)$, is called a local function [2] of A with respect to τ and I is defined as follows: for $A \subseteq X$, $A^*(I, \tau) = \{x \in X : G \cap A \notin I$, for enery $G \in \tau(x)\}$, where, $\tau(x) = \{G \in \tau : x \in G\}$. A kuratowski closure operator $Cl^*()$ for a topology $\tau^*(I, \tau)$ is called *-topology, finer than τ is defined by $Cl^*(A) = A \cup A^*(I, \tau)$ [2]. When there is no chance for confusion, we will simply write A^* for $A^*(I, \tau)$ and τ^* for $\tau^*(I, \tau)$. If I is an ideal on X, then the space (X, τ, I) is called an ideal topological space.

The notion of a nano topology was introduced by Lellis Thivagar and Carmel Richard [3] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also they defined nano closed sets, nano-interior and nano-closure. The concept of nano ideal topological spaces was introduced by Parimala et al. [4] and studied its properties and characterizations. In this paper, we study the relationships between some weak forms of nano open sets in nano topological spaces and some weak forms of nano open sets in nano ideal topological spaces. Also, we point out that the class of nano pre-I-open sets is properly places between the classes of nano I -open and nano pre-open sets. We give a decomposition of nano I-continuity by proving that a function $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ is nano I-continuous if and only if it is nano pre-I-continuous and nano *-I-continuous.

Before entering in our work, we recall the following definitions which are useful in the sequel.

Definition 1.1. *[6] Let U be a nonempty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is* *divided into disjoint equivalence classes. Elements belonging to the some equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space.*

Definition 1.2. *[6] Consider Figure 1*

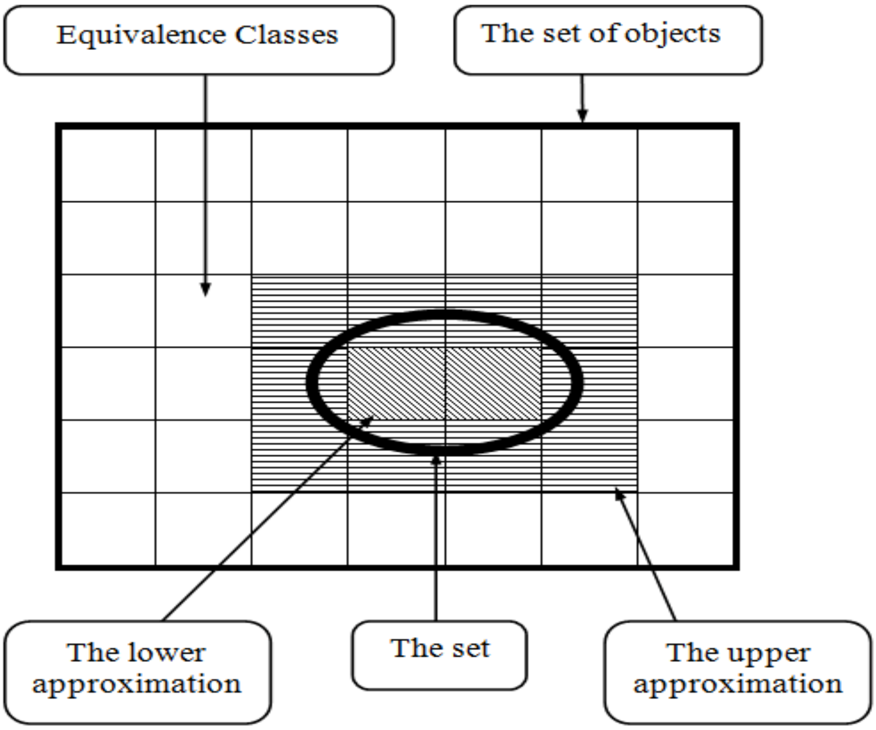

Figure 1: A rough set in a rough approximation space.

Let (U, R) be an approximation space and $X \subseteq U$. Then:

(i) The lower approximation of X with respect to R is the set of all objects , which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$, that is $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by $x \in U$.

(ii) The upper approximation of X with respect to R is the set of all objects , which can be possibly classified as X with respect to R and it is denoted by $H_R(X)$, that is $H_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not $-X$ with respect to R and it is denoted by $B_R(X)$, that is $B_R(X) = H_R(X) - L_R(X)$.

According to Pawlak's definition, X is called a rough set if $H_R(X) \neq L_R(X)$.

Proposition 1.3. *([6]) If (U, R) is an approximation space and $X, Y \subseteq U$, then we have the following properties of Pawlak's rough sets:*

(i) $L_R(X) \subseteq X \subseteq H_R(X)$ (Contraction and Extension).

(ii) $L_R(\phi) = H_R(\phi) = \phi$ (Normality) and $L_R(U) = H_R(U) = U$ (Co-normality).

(iii) $H_R(X \cup Y) = H_R(X) \cup H_R(Y)$ (Addition).

(iv) $H_R(X \cap Y) \subseteq H_R(X) \cap H_R(Y)$.

(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.

(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$ (Multiplication).

(vii) $L_R(X) \subseteq L_R(Y)$ and $H_R(X) \subseteq H_R(Y)$ whenever $X \subseteq Y$ (Monotone)

(viii) $H_R(X^c) = [H_R(X)]^c$ and $L_R(X^c) = [H_R(X)]^c$ where X^c denotes the complement of X in U (Duality)

(ix) $H_R(H_R(X)) = L_R(H_R(X)) = H_R(X)$ (Idempotency).

(x) $L_R(L_R(X)) = H_R(L_R(X)) = L_R(X)$ (Idempotency).

Definition 1.4. *[3] Let U be the universe, R be an equivalence relation on U, then for $X \subseteq U$, $\tau_R(X) = \{U, \phi, L_R(X), H_R(X), B_R(X)\}$ is called the nano topology on U which satisfies the following axioms:*

(i) U and $\phi \in \tau_R(X)$;

(ii) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$;

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X and the pair $(U, \tau_R(X))$ is called a nano topological space. The elements of $\tau_R(X)$ are called nano open sets in U and the complement of a nano open set is called a nano closed set. The elements of $[\tau_R(X)]^c$ being called dual nano topology of $\tau_R(X)$.

Remark 1.5. *If $\tau_R(X)$ is nano topology on U with respect to X, then Lellis Thivagar and Carmel Richard [3] observed that the family $\beta = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Remark 1.6. *Let $(U, \tau_R(X))$ be a nano topological space with respect to X where $X \subseteq U$ and R be an equivalence relation on U. Then U/R denotes the family of equivalence classes of U by R.*

Definition 1.7. *[3] If $(U, \tau_R(X))$ is a nano topological space with respect to X, where $X \subseteq U$ and if $A \subseteq U$, then:*

(i) The nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by $nInt(A)$. That is $nInt(A)$ is the largest nano open subset of A;

(ii) The nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by $nCl(A)$. That is $nCl(A)$ is the smallest nano closed set containing A.

Definition 1.8. *([3], [7], [9]) Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be:*

(i) Nano regular open if $A = nInt(nCl(A))$,

(ii) Nano α-open if $A \subseteq nInt(nCl(nInt(A)))$,

(iii) Nano semi-open if $A \subseteq nCl(nInt(A))$,

(iv) Nano preopen if $A \subseteq nInt(nCl(A))$,

(v) Nano γ-open (or nano b-open) if $A \subseteq nCl(nInt(A)) \cup nInt(nCl(A))$,

(vi) Nano β-open if $A \subseteq nCl(nInt(nCl(A)))$.

The complement of a nano regular open (resp. nano α-open, nano semi-open, nano preopen, nano γ-open, nano β-open) set is called a nano regular closed (resp. nano α-closed, nano semi-closed, nano preclosed, nano γ-closed, nano β-closed) set. The family of all nano semi-open sets of a nano topological space $(U, \tau_R(X))$ is denoted by $NSO(U, X)$.

2 Nano ideal topological spaces

In this section, we shall investigate the nano lacal function in a nano ideal topological space.

Definition 2.1. *[4] Let $(U, \tau_R(X), I)$ be a nano ideal topological space. A set operator $()^*_n : P(U) \rightarrow P(U)$ is called the nano local function. And for a subset $A \subseteq U$, $A^*_n(I, \tau_R(X)) = \{x \in U : G_x \cap A \notin I,$ for every $G_x \in \tau_R(X)\}$ is called the nano local function of A with respect to I and $\tau_R(X)$, we will simply write A^*_n for $A^*_n(I, \tau_R(X))$.*

Example 2.2. *Let $(U, \tau_R(X))$ be a nano topological space with an ideal I on U and for every $A \subseteq U$:*

(i) If $I = \{\phi\}$, then $A^*_n = nCl(A)$,

(ii) If $I = P(U)$, then $A^*_n = \phi$.

The following theorem contains many basic and useful facts concerning the nano local function.

Theorem 2.3. *[5] Let $(U, \tau_R(X))$ be a nano topological space with an ideals I, J on U and A, B be subsets of U, Then the following statements are true:*

(i) If $A \subseteq B$, then $A^*_n \subseteq B^*_n$,

(ii) If $I \subseteq J$, then $A^*_n(J) \subseteq A^*_n(I)$,

(iii) $A^*_n = nCl(A^*_n) \subseteq nCl(A)$, this means, A^*_n is a nano closed subset of $nCl(A)$,

(iv) $(A^*_n)^*_n \subseteq A^*_n$,

(v) $(A \cup B)^*_n = A^*_n \cup B^*_n$,

(vi) $(A \cap B)^*_n \subseteq A^*_n \cap B^*_n$,

(vii) $A^*_n - B^*_n = (A - B)^*_n - B^*_n \subseteq (A - B)^*_n$,

(viii) If $V \in \tau_R(X)$, then $V \cap A^*_n = V \cap (V \cap A)^*_n \subseteq (V \cap A)^*_n$,

(ix) If $E \in I$, then $(A \cup E)^*_n = A^*_n = (A - E)^*_n$.

Theorem 2.4. *Let $(U, \tau_R(X), I)$ be a nono ideal topological space and A, B be subsets of U. Then:*

(i) A^*_n *is a nano closed,*

(ii) $\phi^*_n = \phi$,

(iii) $(U - E)^*_n = U^*_n$, *if* $E \in I$,

(iv) $[U - (A - E)]^*_n = [(U - A) \cup E]^*_n$, *if* $E \in I$.

Definition 2.5. *Let* $(U, \tau_R(X))$ *be a nono topological space with an ideal* I *on* U, *the set operator* nCl^* *is called a nano* **-closure and is defined as:* $nCl^*(A) = A \cup A^*_n$, *for* $A \subseteq X$.

Theorem 2.6. *[4] The set operator* nCl^* *satisfies the following conditions:*

(i) $A \subseteq nCl^*(A)$,

(ii) $nCl^*(\phi) = \phi$ *and* $nCl^*(U) = U$,

(iii) If $A \subseteq B$, *then* $nCl^*(A) \subseteq nCl^*(B)$,

(iv) $nCl^*(A) \cup nCl^*(B) = nCl^*(A \cup B)$,

(v) $nCl^*(nCl^*(A)) = nCl^*(A)$.

Definition 2.7. *([3], [5]) Let* $(U, \tau_R(X), I)$ *be a nono ideal topological space, then* $A \subseteq U$ *is said to be:*

(i) Nano regular I*-open (nano R-I-open) if* $A = nInt(nCl^*(A))$,

(ii) Nano regular I*-closed (nano R-I-closed) if its complement is nano R-I-open,*

(iii) Nano semi I*-open if* $A \subseteq nCl^*(nInt(A))$,

(iv) Nano semi I*-closed if its complement is nano semi* I*-open,*

(v) Nano α*-*I*-open if* $A \subseteq nInt(nCl^*(nInt(A)))$,

(vi) Nano α*-*I*-closed if its complement is nano*α*-*I*-open,*

(vii) Nano pre- I*-open if* $A \subseteq nInt(nCl^*(A))$,

(viii) Nano pre- I*-closed if its complement is nano pre-* I*-open,*

(ix) Nano β*-*I*-open if* $A \subseteq nCl(nInt(nCl^*(A)))$,

(x) Nano β*-*I*-closed if its complement is nano* β*-*I*-open.*

(xi) Nano I-open if $A \subseteq nInt(A_n^*)$.

Definition 2.8. *[4] An ideal I in a nano ideal topological space* $(U, \tau_R(X), I)$ *is called* $\tau_R(X)$*-codense ideal if* $\tau_R(X) \cap I = \{\phi\}$.

Definition 2.9. *[4] A subset A in a nano ideal topological space* $(U, \tau_R(X), I)$ *is said to be:*

(i) Nano $\star$*-dense-in-itself if* $A \subseteq A_n^*$,

(ii) Nano $\star$*-closed if* $A_n^* \subseteq A$,

(iii) Nano $\star$*-perfect if* $A = A_n^*$,

(iv) Nano I-dense if $A_n^* = U$.

3 Nano pre-*I*-open sets

Definition 3.1. *A subset A of a nano ideal topological space* $(U, \tau_R(X), I)$ *is called nano pre-I-open if* $A \subseteq nInt(nCl^*(A))$.

We denote by NPIO$(U, \tau_R(X), I)$ the family of all nano pre-I-open subsets of $(U, \tau_R(X), I)$ or simply write NPIO$(U, \tau_R(X))$ or NPIO(U) when there is no chance for confusion with the ideal. We call a subset $A \subseteq (U, \tau_R(X), I)$ nano pre-I-closed if its complement is nano pre-I-open.

Although nano I-openness and nano openness are independent concepts [[4], Example 4.1], nano pre-I-openness is related to both of them as the following two results show.

Proposition 3.2. *Every nano I-open set is nano pre-I-open.*

Proof. Let $(U, \tau_R(X), I)$ be a nano ideal topological space and $A \subseteq U$ be nano I-open. Then $A \subseteq nInt(A_n^*) \subseteq nInt(A_n^* \cup A) = nInt(nCl^*(A))$. □

Proposition 3.3. *Every nano open set is nano pre-I-open.*

Proof. Let $A \subseteq (U, \tau_R(X), I)$ be nano open. Then $A \subseteq nInt(A) \subseteq nInt(A_n^* \cup A) = nInt(nCl^*(A))$. □

The converse of the above propositions are not necessarily true as shown by the following example.

Example 3.4. *Let* $U = \{a, b, c, d\}$*,* $U/R = \{\{b\}, \{d\}, \{a, c\}\}$ *be the family of equivalence classes of* U *by the equivalence relation* R *and* $X = \{a, d\}$*. One can deduce that* $\tau_R(X) = \{U, \phi, \{d\}, \{a, c\}, \{a, c, d\}\}$*. For* $I = \{\phi, \{d\}\}$*, if* $A = \{a, d\}$*, we have* $A_n^* = \{a, b, c\}$ *and* $nInt(A_n^*) = \{a, c\}$ *and* $nCl^*(A) = A \cup A_n^* = \{a, b, c, d\}$*. So* $nInt(nCl^*(A)) = U$*. Hence* A *is nano pre-I-open but nighter nano I-open nor nano open.*

Our next two results together with Proposition 3.2 and Proposition 3.3 shows that the class of nano pre-I-open sets is properly placed between the classes of nano I-open and nano preopen sets as well as between the classes of nano open and nano preopen sets.

Proposition 3.5. *Every nano pre-I-open set is nano preopen .*

Proof. Let $(U, \tau_R(X), I)$ be a nano ideal topological space and $A \in \text{NPIO}(U)$. Then $A \subseteq nInt(nCl^*(A)) = nInt(A_n^* \cup A) \subseteq nInt(nCl(A) \cup A) = nInt(nCl(A))$. □

Remark 3.6. *In the nano ideal topological space* $(U, \tau_R(X), I)$*, several forms of weak nano open sets defined above have the following implications as in Figure 2.*

However the converses of these implications of Figure 2 are not true in general by Example 2.1, Example 2.2 [7] and the following examples.

Example 3.7. *Let* $U = \{a, b, c, d\}$*,* $U/R = \{\{b\}, \{d\}, \{a, c\}\}$ *be the family of equivalence classes of* U *by the equivalence relation* R *and* $X = \{a, d\} \subseteq U$*. One can deduce that* $\tau_R(X) = \{U, \phi, \{d\}, \{a, c\}, \{a, c, d\}\}$*.*

(a) For $I = \{\phi, \{d\}\}$*. Then:*

(i) $A = \{a, d\}$ *is nano pre-I-open but nighter nano I-open nor nano* α*-I-open.*

(ii) $B = \{a, b, c\}$ *is nano semi-I-open but not nano* α*-I-open.*

(iii) Also, B *is nano* β*-I-open but not nano pre-I-open.*

(iv) $C = \{b, d\}$ *is nano semi-open but not nano semi-I-open.*

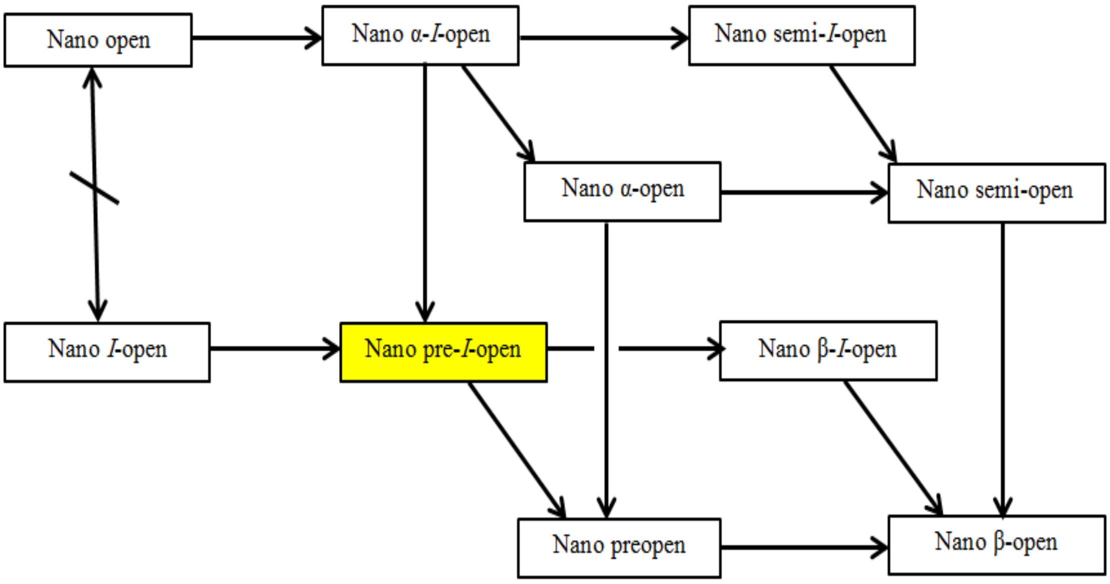

Figure 2: Comparison between weak nano open sets

(b) For $I = \{\phi, \{a\}\}$. Then: $A = \{a, d\}$ is nano preopen but not nano pre-I-open.

(c) For $I = \{\phi, \{a\}, \{d\}, \{a, d\}\}$. Then: $A = \{a, b, d\}$ is nano β-open but not nano β-I-open.

Theorem 3.8. *For a nano ideal topological space $(U, \tau_R(X), I)$ and $A \subseteq U$, we have:*

(i) If $I = \{\phi\}$, then A is nano pre-I-open if and only if A is nano preopen.

(ii) If $I = P(U)$, then A is nano pre-I-open if and only if $A \in \tau_R(X)$.

(iii) If $I \subseteq I_n$, then A is nano pre-I-open if and only if A is nano preopen, where I_n denote the ideal of nowhere dense sets.

Proof. (i) Necessity is shown in Proposition 3.5. For sufficiency note that in the case of the minimal ideal $A_n^* = nCl(A)$.

(ii) Necessity, if $A \in \text{NPIO}(U)$, then $A \subseteq nInt(nCl^*(A)) = nInt(A_n^* \cup A) = nInt(A \cup \phi) = nInt(A)$. Sufficiency, is given by Proposition 3.3.

(iii) By Proposition 3.5, we need to show only sufficiency. Note that the nano local function of A with respect to I_n and τ can be given by $A_n^*(I_n) = nCl(nInt(nCl(A)))$.

Thus A is nano pre-I-open if and only if $A \subseteq nInt(A \cup nCl(nInt(nCl(A))))$. Assume that A is nano preopen. Since always $nInt(nCl(A)) \subseteq A \cup nCl(nInt(nCl(A)))$, then $A \subseteq nInt(A \cup nCl(nInt(nCl(A))))$ or equivalently A is nano pre-I-open. □

Remark 3.9. *The intersection of two nano pre-I-open sets need not be nano pre-I-open.*

Lemma 3.10. *Let $(U, \tau_R(X), I)$ be a nano ideal topological space and let $A \subseteq U$. Then $G \in \tau_R(X)$ implies $G \cap A^*_n = G \cap (G \cap A)^*_n \subseteq (G \cap A)^*_n$.*

Proof. If $V \in \tau_R(X)$ and $x \in G \cap A^*_n$. Then $x \in G$ and $x \in A^*_n$. Let H be any nano open set containing x. Then $H \cap G \in \tau_R(X)$ and $H \cap (G \cap A) = (H \cap G) \cap A \notin I$. This shows that $x \in (G \cap A)^*_n$ and hence $G \cap A^*_n \subseteq (G \cap A)^*_n$. Hence $G \cap A^*_n \subseteq G \cap (G \cap A)^*_n$. By (i), $(G \cap A)^*_n \subseteq A^*_n$ and $G \cap A^*_n \supseteq G \cap (G \cap A)^*_n$. Therefore $G \cap A^*_n = G \cap (G \cap A)^*_n \subseteq (G \cap A)^*_n$. □

Proposition 3.11. *Let $(U, \tau_R(X), I)$ be a nano ideal topological space with $\triangle$ an arbitrary index set. Then:*

(i) If $\{A_\alpha : \alpha \in \triangle\} \in NPIO(U)$, then $\bigcup\{A_\alpha : \alpha \in \triangle\} \in NPIO(U)$.

(ii) If $A \in NPIO(U)$ and $G \in \tau_R(X)$, then $A \cap G \in NPIO(U)$.

Proof. (i) Since $\{A_\alpha : \alpha \in \triangle\} \in$ NPIO(U), then $A_\alpha \subseteq nInt(nCl^*(A_\alpha))$ for every $\alpha \in \triangle$. Thus $\bigcup\limits_{\alpha\in\triangle} A_\alpha \subseteq \bigcup\limits_{\alpha\in\triangle} nInt(nCl^*(A_\alpha)) \subseteq nInt(\bigcup\limits_{\alpha\in\triangle} nCl^*(A_\alpha)) = nInt(\bigcup\limits_{\alpha\in\triangle} (A^*_{n\alpha} \cup A_\alpha)) = nInt((\bigcup\limits_{\alpha\in\triangle} A^*_{n\alpha}) \cup (\bigcup\limits_{\alpha\in\triangle} A_\alpha)) \subseteq nInt((\bigcup\limits_{\alpha\in\triangle} A_\alpha)^*_n \cup (\bigcup\limits_{\alpha\in\triangle} A_\alpha)) = nInt(nCl^*(\bigcup\limits_{\alpha\in\triangle} A_\alpha))$

(ii)By assumption, $A \subseteq nInt(nCl^*(A))$ and $G \subseteq nInt(G)$. Then applying Lemma 3.10, $A \cap G \subseteq nInt(nCl^*(A)) \cap nInt(G) \subseteq nInt(nCl^*(A) \cap G) = nInt((A^*_n \cup A) \cap G) = nInt((A^*_n \cap G) \cup (A \cap G)) \subseteq nInt((A \cap G)^*_n \cup (A \cap G)) = nInt(nCl^*(A \cap G))$. □

Corollary 3.12. *(i) The intersection of an arbitrary family of nano pre-I-closed sets is nano pre-I-closed.*

(ii) The union of a nano pre-I-closed set and a nano closed set is nano pre-I-closed.

4 Nano I-continuous functions

In this section, we define and study the nano I-continuous functions and study some of its properties.

Definition 4.1. *A function $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$ is called nano I-continuous function if for every $G \in \tau_{R'}(Y)$, $f^{-1}(G) \in NIO(U, \tau_R(X))$.*

Recall that $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$ nano precontinuous if the inverse image of every nano open set in V is nano preopen in U.

Remark 4.2. *From the above definitions one may notice that:*

Nano* I*-continuity* $\Rightarrow$ *Nano precontinuity

and the converse is not true in general as shown by Example 4.3.

Example 4.3. *Let $U = \{a, b, c, d\}$ be the universe, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ be the family of equivalence classes of U by the equivalence relation R and $X = \{a, b\} \subseteq U$. One can deduce that $\tau_R(X) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Let $V = \{x, y, z, w\}$ be the universe, $V/R' = \{\{x\}, \{w\}, \{y, z\}\}$ be the family of equivalence classes of V by the equivalence relation R' and $Y = \{x, z\} \subseteq V$. One can deduce that $\tau_{R'}(Y) = \{V, \phi, \{x\}, \{y, z\}, \{x, y, z\}\}$. Define $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$ as $f(a) = x$ $f(b) = y$, $f(c) = w$, $f(d) = z$. For $I = \{\phi, \{a\}, \{b\}, \{a, b\}\}$, f is nano precontinuous but not nano I-continuous.*

Theorem 4.4. *For a function $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$, the following are equivalent,*

(i) f is nano I-continuous.

(ii) For each $x \in U$ and each $G \in \tau_{R'}(Y)$ containing $f(x)$, there exists $W \in NIO(U)$ containing x such that $f(W) \subseteq G$.

(iii) For each $x \in U$ and each $G \in \tau_{R'}(Y)$ containing $f(x)$, $(f^{-1}(G))^*_n$ is a nano neighborhood of x.

Proof. (i)$\Rightarrow$ (ii): Since $G \in \tau_{R'}(Y)$ containing $f(x)$, then by (i), $f^{-1}(G) \in$ NIO(U), by putting $W = f^{-1}(G)$ which containing x. Therefore $f(W) \subseteq G$.

(ii)⇒ (iii): Since $G \in \tau_{R'}(Y)$ containing $f(x)$, then by (ii), there exists $W \in \text{NIO}(U)$ containing x such that $f(W) \subseteq G$. So, $x \in W \subseteq nInt(W)^*_n \subseteq nInt(f^{-1}(G))^*_n \subseteq (f^{-1}(G))^*_n$. Hence $(f^{-1}(G))^*_n$ is a nano neighborhood of x.
(iii)⇒ (i): Obvious. □

5 A decomposition of nano I-continuity

. In this section, we study the composition of nano I-continuous functions with another types of nano continuity.

Definition 5.1. *A function $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$ is called:*

(i) Nano $*$-I-continuous if the inverse image of every nano open set in $(V, \tau_{R'}(Y))$ is nano $*$-dense-in-itself in $(U, \tau_R(X), I)$.

(ii) Nano pre-I-continuous if for every $V \in \tau_{R'}(Y)$, $f^{-1}(V) \in NPIO(U, \tau_R(X))$.

In the notion of Proposition 3.3, we have the following result.

Proposition 5.2. *Every nano continuous function $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$ is nano pre-I-continuous.*

The converse is not true in general as shown in the following example.

Example 5.3. *Let $U = \{a, b, c, d\}$ be the universe, $U/R = \{\{a\}, \{d\}, \{b, c\}\}$ be the family of equivalence classes of U by the equivalence relation R and $X = \{a, b\}$ and let $V = \{x, y, z, w\}$ be the universe, $V/R' = \{\{x\}, \{z\}, \{y, w\}\}$ be the family of equivalence classes of V by the equivalence relation R' and $Y = \{x, y\}$. One can deduce that $\tau_R(X) = \{U, \phi, \{a\}, \{d\}, \{a, d\}\}$ and $\tau_{R'}(Y) = \{V, \phi, \{x\}, \{y, w\}, \{x, y, w\}\}$. Define $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$ as $f(a) = y = f(b)$, $f(c) = z$, $f(d) = w$. For $I = \{\phi, \{d\}\}$. One can deduce that f is nano pre-I-continuous, but not nano continuous.*

Due to Proposition 3.2, we have the next result.

Proposition 5.4. *Every nano I-continuous function $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$ is nano pre-I-continuous.*

The reverse is again not true as the following example shows.

Example 5.5. *In Example 5.3, f is nano pre-I-continuous, but not nano I-continuous.*

Proposition 5.6. *Every nano pre-I-continuous function $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$ is nano precontinuous.*

Proof. Follows directly by Proposition 3.5. □

Theorem 5.7. *For a function $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$, the following conditions are equivalent:*

(i) f is nano pre-I-continuous.

(ii) For each $x \in U$ and each $G \in \tau_{R'}(Y)$ containing $f(x)$, then there exists $W \in$ NPIO(U) containing x such that $f(W) \subseteq G$.

(iii) For each $x \in U$ and each $G \in \tau_{R'}(Y)$ containing $f(x)$, $Cl_n^*(f^{-1}(G))$ is a nano neighbourhood of x.

(iv) The inverse image of each nano closed set in $(V, \tau_{R'}(Y))$ is nano pre-I-closed.

Proof. (i) ⇒ (ii): Let $x \in U$ and $G \in \tau_{R'}(Y)$ such that $f(x) \in G$. Put $W = f^{-1}(G)$. By (i), W is nano pre-I-open and clearly $x \in W$ and $f(W) \subseteq G$.

(ii) ⇒ (iii) : Since $G \in \tau_{R'}(Y)$ and $f(x) \in G$, then by (ii), there exists $W \in$ NPIO(U) containing x such that $f(W) \subseteq G$. Thus $x \in W \subseteq nInt(nCl^*(W)) \subseteq nInt(nCl^*(f^{-1}(G))) \subseteq nCl^*(f^{-1}(G))$. Hence $nCl^*(f^{-1}(G))$ is nano neighbourhood of x.

(iii) ⇒ (i) and (i) ⇔ (iv) are obvious. □

Remark 5.8. *The composition of two nano pre-I-continuous functions need not be always nano pre-I-continuous. Because of each nano pre-I-open is not nano open as shown in Figure 2.*

Theorem 5.9. *Let $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y), J)$ and $g : (V, \tau_{R'}(Y), J) \longrightarrow (W, \tau_{R''}(Z))$ be two functions, where I and J are ideals on U and V, respectively, then:*

(i) $g \circ f$ is nano pre-I-continuous, if f is a nano pre-I-continuous and g is nano continuous.

(ii) $g \circ f$ is nano precontinuous, if g is nano continuous and f is nano pre-I-continuous.

Proof. Obvious. □

Now, we give a decomposition of nano I-openness.

Theorem 5.10. *For a subset $A \subseteq (U, \tau_R(X), I)$ the following conditions are equivalent:*

(i) A is nano I-open.

(ii) A is nano pre-I-open and nano $*$-dense-in-itself.

Proof. (i) $\Rightarrow$ (ii):By Proposition 3.3 every nano I-open set is nano pre-I-open. On the other hand $A \subseteq nInt(A_n^*) \subseteq A$ which shows that A is nano $*$-dense-in-itself.

(ii) $\Rightarrow$ (i) By assumption $A \subseteq nInt(nCl^*(A)) = nInt(A_n^* \cup A) = nInt(A_n^*)$ or equivalently A is nano I-open. □

In what follow we try do decompose nano I-continuity.

Theorem 5.11. *For a function $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$, the following conditions are equivalent:*

(i) f is nano I-continuous.

(ii)f is nano pre-I-continuous and f is nano $*$-I-continuous.

6 Conclusion

We hope that this paper is just a beginning of a new structure. It will inspire many to contribute to the cultivation of nano ideal topology in the field of mathematical structures of nano approximations. We define a new definition of pre-I-open set and investigate some of its properties. Also, we give a comparison between pre-I-open set and some other types near nano openness. Some numerous types of nano I-continuous functions and study some of its properties. Finally, we study the composition of nano I-continuous functions with another types of nano continuity.

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