A Theoretical Study Of The Effect Of A Laser Mode With Different Coherent Angles On The Motion Of Atomic Vapors

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Abstract

We follow theoretically the motion of the sodium atoms in vapor state under the influence of a laser mode in (1 + 1) D, which is achieved via different optical filters. In the Dirac interaction representation, the equations of motion are represented via the Bloch form together with the Pauli operators to find the elements of the density matrix of the system. The immergence of the principle of coherence in varying the angles of the laser mode permits the evaluation of the average force affecting the atoms acceleration or deceleration; and hence the corresponding velocities and temperatures are investigated. The atomic vapor is introduced in a region occupied by a heat bath presented by the laser field, such that the state of the atomic vapor is unstable inside the system due to the loss or gain of its kinetic energy to or from the laser field. This instability is studied through finding the eigenvalues of the system's entropy. Resorting to the assumption of Boten, Kazantsev and Pusep, who issued a coupling between the mean numbers of photons in terms of time, allows the evaluation of the rate of entropy production of the system under study. A set of figures illustrating the dynamics of the problem is presented.

Keywords: Laser pressure on atoms; Dirac representation; Coherent states; Spontaneous emission; Irreversible statistical mechanics.
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Introduction

The prediction of the mechanical effects of laser light on the neutral atoms dates back to Ashkin **[1]** and kazantsev **[2]**. More specifically, in recent years atomic beams have been laser-cooled and trapped, obtaining a narrowing of atomic lines and slowing down of atomic velocity. In the case of a plane monochromatic laser – traveling wave in the same direction as a two level atomic beam, Kazantsev **[2]** solved the equation of motion for the atomic density matrix with a semi classical Hamiltonian, and the field wave vector is taken in complex form that the resulting force lies in the complex plane, its real part takes the direction of the atomic beam and accelerates them, it was termed by the spontaneous radiation pressure force following Ashkin **[1].** The imaginary part or the transverse component is the gradient or the dipole force which is in direct proportionality with the field intensity. Interaction representation has a long history in the study of atom dynamics in laser waves; the Dirac representation of a two-level atom and a nearly resonant light field had received a theoretical tackling in **[4]**. A work conducted by Letokhov and Minogin was based on the semi classical approach, such as in **[5]**, who presented a quantum treatment based on the Schrödinger perception of the motion of atoms in a resonant light field. The interaction Hamiltonian is taken in the dipole and rotating wave approximations. Coherent states were first studied by Schrödinger in 1926, and were rediscovered by Klauder , Glauber , and Sudarshan at the beginning of the 1960's, they described the specific role of coherent states played in quantum radiation fields **[6]**. In reference **[7],** the acceleration of atoms by laser using the Landau-Lifshitz (LL) equation was **a** discussed. Since laser cooling decreases the temperature of a sample of atoms, there is less disorder and therefore less entropy. This seems to conflict with the second law of thermodynamics, which requires the entropy of a closed system to always increase with time. The explanation lies in the consideration of the fact that in laser cooling, the atoms do not form a closed system. Instead, there is always a flow of laser light with low entropy into the system and fluorescence with high entropy out of it. The decrease of entropy of the atoms is accompanied by a much larger increase in entropy of the light field. Entropy considerations for a laser beam are far from trivial, but recently it has been shown that the entropy lost by the atoms is many orders of magnitude smaller than the entropy gained by the light field**.[8]**

The concept of entropy plays an important role in our understanding of complex physical systems . The study of the entropy in quantum systems was begun by von Neumann in 1932**.** The quantum entropy for a density operator was defined by von Neumann about 20 years before the Shannon entropy appeared. The quantum dynamical entropy (QDE) was studied in **[9,10].** In 1976 , Pusep has investigated the quantum features of the acceleration of an atom in the homogenous field of a traveling monochromatic wave. The atoms are accelerated as a result of absorption of photons of the traveling wave and of the spontaneous emission of a spherical wave. The improvement of the methodology utilized by Kazantsev**[11]**, consists of the replacement of the classical description of the translational motion of the atom by a consistent quantum mechanical description.

Indeed, upon absorbing a photon from the light flux and spontaneously emitting a spherical wave, the atoms acquires the momentum hk in the direction of propagation of the wave within a cycle of duration γ^{-1}. Despite that the spherical wave correspond to the classical limit, from the quantum standpoint each elementary event of photon emission bears away a momentum equals in magnitude to hk in an arbitrary direction.

When the photon number n is characterized by a Poisson distribution , Botin and kazanstev **[2]** and just after them Pusep **[3]** suggested that there is a linear relationship between the mean number of scattered photons and the time of laser-atom interaction in the form $\bar{n} = \gamma\, t\,/2$ if the spontaneous emission is strong, while $\bar{n} = 2\,G\,\gamma\,t$ if the the spontaneous emission is weak, where γ^{-1} is the life time of the atomic levels. Since for $\bar{n} >> 1$ the distribution of $\bar{n}$ is well approximated by a Gaussian, while the distribution to which emission of photons gives rise in the case $n >> 1$ is also Gaussian. In 2016, Andrede et-al **[12]** studied the entropy of a quantized field in interaction with a two- level atom in a pure state when the field is initially in a mixture of two number states. In this paper, a theoretical study discusses the effects on a neutral atomic vapor by a plane of monochromatic laser traveling wave in different coherent angles. The pressure force, the velocity and temperature of atoms estimated.

The physical problem

We consider a plane traveling wave, a single laser mode, of frequency ω as propagating in the z-direction with wave vector ($\vec{K} = \pm k\, \vec{e}_z$), while a beam of vapor of Na atoms moves in the $(+ve)$ z-direction. The role of the coherent state is played by introducing suitable optical filters inducing different angles $\emptyset$ to affect the state of motion of atoms making either the acceleration or deceleration, this is followed by a study which discusses the statistical nature of the quantum system via entropy and entropy production.

The Two-Level Model is employed to represent our problem. An atom with only two energy eigenvalues are described as a two-dimensional states space spanning between the two energy eigenstates $|2\rangle$ and $|1\rangle$. The two states constitute a complete orthonormal system [13]. The corresponding energy eigenvalues are E_1 and E_2. The energy levels of the atom is in a coordinate frame rotating with frequency ω_0.

We use a type of laser with frequency ω approaching to the transition frequency ω_0 of sodium atoms. $\omega \sim \omega_0$, $\omega_0 = (\omega_1 - \omega_2)$; the atomic levels $|1\rangle$ *and* $|2\rangle$ are coupled by the light induced transitions, are separated in coordinate systems by a difference $\hbar\Delta, \Delta = \omega_0 - \omega$

The real value of the density matrix formalism for atom-light interactions is its ability to deal with open systems. The reason is that; the closed system of atom plus laser light that can be described by Schrödinger wave functions and is thus in a pure state, undergoes evolution to a ‘‘mixed’’ state by virtue of the spontaneous emission.[8]

The interaction representation

The interaction Hamiltonian is taken in the dipole and rotating wave approximations in the framework of the interaction representation[14];

$$\hat{H}_I = i\,\hbar g\,\{\hat{\pi}^{\dagger}\hat{a}e^{i\theta} - \hat{a}^{\dagger}\hat{\pi}e^{-i\theta}\} \tag{1}$$

Where:

- $\theta = \Delta t + \vec{K}.\vec{R} = \Delta t - kz \quad , \Delta = \omega_0 - \omega > 0$,

where ω_0 for the atoms and ω for the field , $\vec{R} = z\, \vec{e}_z$ is the atom position vector;

- g is Rabi frequency.
- $\{\hat{\pi}^{\dagger} = |2\rangle\langle 1| \ ,\ \hat{\pi} = |1\rangle\langle 2|\ \}$ are the transition operators for atoms.
- $\hat{a}^{\dagger}, \hat{a}$ are the boson operators.

We present the equation of motion of the matrix elements of the density matrix operator as $\hat{\rho}_I(t)$. The interaction of the two-level atom with the quantized electric field of an electromagnetic wave is defined by the Bloch equations:

$$\frac{d\rho_{ij}}{dt} = -\frac{i}{\hbar}\left\langle i \left| \left[\hat{H}_I , \rho_I \right] \right| j \right\rangle - \Gamma_{ij}(\rho_{ij} - \rho_{ij}^0) \ , i , j = 1, 2 \tag{2}$$

Where:

- $\Gamma_{11} = \Gamma_{22} = 2\gamma$, are the spontaneous decay rates for the two metastable levels.
- $\Gamma_{12} = \Gamma_{21} = \gamma$, are the rates of relaxation of the off- diagonal matrix element.
- At t=0, we assume that the initial populations of the lower and upper states are
  $\rho_{11}^0 = 1, and\ \rho_{22}^0 = \rho_{12}^0 = \rho_{21}^0 = 0$.

Following [12] from (1) and (2), using the initial conditions for i=j=1; the equations of motion are respectively (note that: $\bullet \equiv \frac{d}{dt}$)

$$\dot{\rho}_{11} = -2\gamma - 2\gamma\,\rho_{11} + g\,\{\hat{a}^\dagger \rho_{21} e^{-i\theta} + \rho_{12}\hat{a}\, e^{i\theta}\} \tag{3.a}$$

Similarly , for j =i =2

$$\dot{\rho}_{22} = -2\gamma\,\rho_{22} + g\,\{\hat{a}\rho_{12}\, e^{i\theta} + \rho_{21}\hat{a}^\dagger\, e^{-i\theta}\} \tag{3.b}$$

for j =2, i =1

$$\dot{\rho}_{12} = -\gamma\,\rho_{12} + g\,\{-\hat{a}^\dagger \rho_{22} + \rho_{11}\hat{a}^\dagger\} e^{-i\theta} \tag{3.c}$$

for j =1 ,i =2

$$\dot{\rho}_{21} = -\gamma\,\rho_{21} + g\,\{-\rho_{22}\,\hat{a} + \hat{a}\,\rho_{11}\} e^{i\theta} \ . \tag{3.d}$$

The density matrix satisfies the conditions for mixed state.

$$Tr\ \rho_I = 1\ ,\ \rho_{12} = \rho_{21}^\dagger \quad ,\ \rho_I^2 \neq \rho_I \ .$$

Resorting to the coherent state $\left|\alpha\right\rangle$ of the laser mode, to obtain the value of density matrix elements according to which is defined as an eigenstate of the amplitude operator, one of the annihilation operators $\{\hat{a}\}$, with eigenvalues $\{\alpha\}$ **[15]**. The operators $\{\hat{a}\}$ are non-Hermitian , and the phase angle ϕ describes the wave aspect of the coherent state $\alpha = |\alpha| e^{i\phi} \in C$, where as α is a complex number, which corresponds to the complex wave amplitude in classical optics. Thus the coherent states are wave-like states of the electromagnetic oscillators. The coherence angles ϕ are the phase of the electromagnetic field (laser) which affects the atoms.

Although the coherent states are not orthogonal, it is possible to expand them in terms of a complete set of states. The completeness relation for the coherent states **[16];** and the following properties hold true:

$$\frac{1}{\pi}\int d^2\alpha \left|\alpha\right\rangle\left\langle\alpha\right| = 1, \bar{\dot{\rho}}_{ij} = \left\langle\alpha\right|\dot{\rho}_{ij}\left|\alpha\right\rangle, \hat{a}\left|\alpha\right\rangle = \alpha\left|\alpha\right\rangle\ ,\ \left\langle\alpha\right|\hat{a}^\dagger = \left\langle\alpha\right|\alpha^*\ ,\ \left\langle\alpha|\alpha\right\rangle = 1\ (*).$$

so that applying (*) to both sides of equations (3) to get

$$\bar{\dot{\rho}}_{11} = 2\gamma - 2\gamma\,\overline{\rho}_{11} - g\,\{\alpha^*\,\overline{\rho}_{21}\, e^{-i\theta} + \alpha\,\overline{\rho}_{12}\, e^{i\theta}\} \tag{4.a}$$

$$\bar{\dot{\rho}}_{22} = -2\gamma\,\overline{\rho}_{22} + g\,\{\alpha^*\,\overline{\rho}_{21}\, e^{-i\theta} + \alpha\,\overline{\rho}_{12}\, e^{i\theta}\} \tag{4.b}$$

$$\bar{\dot{\rho}}_{12} = -\gamma\,\overline{\rho}_{12} + g\,\alpha^*\{\overline{\rho}_{22} - \overline{\rho}_{11}\} e^{-i\theta} \tag{4.c}$$

$$\bar{\dot{\rho}}_{21} = -\gamma\,\overline{\rho}_{21} + g\,\alpha\{-\overline{\rho}_{22} + \overline{\rho}_{11}\} e^{i\theta} \tag{4.d}$$

Pauli operators

The Pauli matrices are a set of three 2 × 2 complex matrices which are Hermitian and unitary [16].

$$\hat{\sigma}_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \hat{\rho}_{12} + \hat{\rho}_{21} \tag{5.a}$$

$$\hat{\sigma}_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ i & 0 \end{bmatrix} + \begin{bmatrix} 0 & -i \\ 0 & 0 \end{bmatrix} = i\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} - i\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = i(-\hat{\rho}_{12} + \hat{\rho}_{21}) \tag{5.b}$$

$$\hat{\sigma}_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \hat{\rho}_{11} - \hat{\rho}_{22} \tag{5.c}$$

And

$$\hat{\rho}_{11} + \hat{\rho}_{22} = \hat{1} \tag{5.d}$$

- The determinants and traces of the Pauli matrices are:

$$\det \sigma_i = -1 \ , \ Tr\, \sigma_i = 0 \tag{6}$$

- The Pauli vector is defined by

$$\underline{\sigma} = \sigma_x \hat{x} + \sigma_y \hat{y} + \sigma_z \hat{z} \tag{7}$$

Then using coherent state $|\alpha\rangle$ with equations (5.1), (5.2), (5.3) and (5.4)

$$\bar{\sigma}_x = \bar{\sigma}_1 = (\bar{\rho}_{12} + \bar{\rho}_{21}) \tag{8.a}$$

$$\bar{\sigma}_y = \bar{\sigma}_2 = i\,(\bar{\rho}_{21} - \bar{\rho}_{12}) \tag{8.b}$$

$$\bar{\sigma}_z = \bar{\sigma}_3 = (\bar{\rho}_{11} - \bar{\rho}_{22}) \tag{8.c}$$

$$1 = (\bar{\rho}_{11} + \bar{\rho}_{22}) \tag{8.d}$$

Where σ_x *and* σ_y are Hermitian operators.

And we have also, if we differentiate and take the value with coherent state:

$$\dot{\bar{\sigma}}_x = (\dot{\bar{\rho}}_{12} + \dot{\bar{\rho}}_{21}) \tag{9.a}$$

$$\dot{\bar{\sigma}}_y = i\,(\dot{\bar{\rho}}_{21} - \dot{\bar{\rho}}_{12}) \tag{9.b}$$

$$\dot{\bar{\sigma}}_z = (\dot{\bar{\rho}}_{11} - \dot{\bar{\rho}}_{22}) \tag{9.c}$$

$$0 = (\dot{\bar{\rho}}_{11} + \dot{\bar{\rho}}_{22}) \tag{9.d}$$

Using (4.1), (4.2), (4.3) and (4.4) we have:

$$\dot{\bar{\sigma}}_x = -\gamma\,\sigma_x + 2g\,|\alpha|\cos(\theta+\phi)\,\bar{\sigma}_z \tag{10.a}$$

$$\dot{\bar{\sigma}}_y = -\gamma\,\sigma_y - 2g\,|\alpha|Sin(\theta+\phi)\,\bar{\sigma}_z \tag{10.b}$$

$$\dot{\bar{\sigma}}_z = -2\gamma - \gamma\,\bar{\sigma}_z - 2g\,|\alpha|\cos(\theta+\phi)\,\bar{\sigma}_x - 2g\,|\alpha|\sin(\theta+\phi)\,\bar{\sigma}_y \tag{10.c}$$

The general form of these equations (8.1), (8.2), (8.3), and (8.4) is

$$\dot{\bar{\sigma}} = M\,\bar{\sigma} + Y \tag{10.d}$$

Where the matrix representation is

.

$$\bar{\sigma}=\begin{bmatrix}\bar{\sigma}_x\\ \bar{\sigma}_y\\ \bar{\sigma}_z\end{bmatrix}, Y=\begin{bmatrix}0\\0\\2\gamma\end{bmatrix}, M=\begin{bmatrix}-\gamma & 0 & 2g\,i\,|\alpha|\sin(\theta+\phi)\\ 0 & \gamma & -2g\,i\,|\alpha|\cos(\theta+\phi)\\ -2g\,|\alpha|\cos(\theta+\phi) & -2g\,|\alpha|\sin(\theta+\phi) & -2\gamma\end{bmatrix}.$$

The steady state solution is taken into consideration such that equation (10.d) will be:

$\dot{\bar{\sigma}}=0$, implies $\bar{\sigma}=-M^{-1}Y$

$|M|=-2\gamma^3-4g^2\,\bar{n}\,\gamma\neq 0$

$$M^{-1}=\frac{1}{|M|}\begin{bmatrix}2\gamma^2+4g^2\,\bar{n}\sin^2(\theta+\phi) & 4g^2\,\bar{n}\cos(\theta+\phi)\sin(\theta+\phi) & 2g\,\gamma|\alpha|\cos(\theta+\phi)\\ 4i\,g\,\bar{n}\cos^2(\theta+\phi) & 2\gamma^2+4g^2\,\bar{n}\cos^2(\theta+\phi) & -2g\,\gamma|\alpha|\sin(\theta+\phi)\\ -2g\,\gamma|\alpha|\cos(\theta+\phi) & 2g\,\gamma|\alpha|\sin(\theta+\phi) & \gamma^2\end{bmatrix}$$

Then Pauli operators in terms of the coherent states are:

$$\bar{\sigma}_x=\frac{-1}{|M|}4g\,\gamma^2\,|\alpha|\cos(\theta+\phi)\quad=\bar{\rho}_{12}+\bar{\rho}_{21}\quad, \tag{11.a}$$

$$\bar{\sigma}_y=\frac{-1}{|M|}(-4g\,\gamma^2\,|\alpha|\sin(\theta+\phi))=i\,(\bar{\rho}_{12}-\bar{\rho}_{21})\quad. \tag{11.b}$$

$$\bar{\sigma}_z=\frac{-1}{|M|}(2\gamma^3)\quad=\bar{\rho}_{11}-\bar{\rho}_{22}\quad, \tag{11.c}$$

provided that

$$1\quad=\bar{\rho}_{11}+\bar{\rho}_{22}\quad. \tag{11.d}$$

Four cases lie in the scope of this study:-

The first: the traveling wave in the opposite direction of sodium atoms with detuning $\Delta>0$.

The second: the traveling wave in the same direction of sodium atoms with detuning $\Delta>0$.

The third: the traveling wave in the opposite direction of sodium atoms with detuning $\Delta<0$.

The forth: the traveling wave in the same direction of sodium atoms with detuning $\Delta<0$.

The density matrix elements under the effect of the coherence state are

$$\bar{\rho}_{11}=\frac{1+0.5G\bar{n}}{1+G\bar{n}}\ ,\ \bar{\rho}_{22}=\frac{0.5G\bar{n}}{1+G\bar{n}}\ ,\ \bar{\rho}_{12}=\frac{g\,\alpha^* e^{-i\theta}}{\gamma(1+G\,\bar{n})}\ ,\ \bar{\rho}_{21}=\frac{g\,\alpha e^{i\theta}}{\gamma(1+G\,\bar{n})}\ . \tag{12}$$

Where $G=2\left(\frac{g}{\gamma}\right)^2$ is the saturation parameter.

The investigation of the problem in terms of the continuous photon numbers

Recalling that, spontaneous emission causes the state of the system to evolve from a pure state into a mixed state and so the density matrix is needed to describe it. Spontaneous emission is an essential ingredient for the dissipative nature of the optical forces.[8]

Thus the stimulated transitions in the field of a running wave do not contribute to the mean force of the light pressure.
We shall continue our investigations in two limiting cases of rapid and slow spontaneous emission:

The first case

The traveling wave propagates in the $(-ve)$ z-direction ($\vec{K} = -k\,\vec{e}_z$), the density matrix elements have the same values with $\theta_1 = \Delta t - kz$; $\Delta > 0$.

Using the completeness relation in (*);
The density matrix elements therefore are**:**

$$\hat{\rho}_{11} = \frac{1+0.5G\,\hat{a}^{\dagger}\hat{a}}{1+G\,\bar{n}} \;,\; \hat{\rho}_{22} = \frac{0.5G\,\hat{a}^{\dagger}\hat{a}}{1+G\,\bar{n}} \;,\; \hat{\rho}_{12} = \frac{g\,\hat{a}^{\dagger}\,e^{-i\theta_1}}{\gamma(1+G\,\bar{n})} \;,\; \hat{\rho}_{21} = \frac{g\,\hat{a}\,e^{i\theta_1}}{\gamma(1+G\,\bar{n})} \;. \qquad (13)$$

The Force Acting On Atoms

We will use an Electromagnetic wave (laser), where the force acting on a two-level atom in a resonance light field, it can be estimated as follows: in the field of a strong running wave, the atom absorbs a photon from a light beam and acquire the momentum $\hbar k$ of photon [**16**].
The Hamiltonian in matrix representation is

$$\hat{H}_I = \begin{bmatrix} 0 & i\hbar g\,\hat{a}\,e^{i\theta_1} \\ -i\hbar g\,\hat{a}^{\dagger}\,e^{-i\theta_1} & 0 \end{bmatrix} \;, \theta_1 = \Delta t - kz$$

The relation of the optical force acting on atoms**: [17]**

$$F = -\left\langle \frac{\partial H}{\partial z} \right\rangle = -Tr\{\rho_I \frac{\partial H}{\partial z}\}$$

$$F = -\frac{1}{4}\hbar k\,G\gamma\{(1-i) + \hat{a}^2 e^{2i\theta_1} - \hat{a}^{\dagger 2}\,e^{-2i\theta_1}\} \;. \qquad (14)$$

The value of the force with respect to the coherent state $|\alpha\rangle$ while using the relation (**) and the expansion of the exponential function is:
When ,

$$\bar{F} = \langle \alpha | F | \alpha \rangle = \hbar k\;\gamma\;W_1;\; W_1 = \frac{G\bar{n}}{1+G\,\bar{n}}\cos 2(\theta_1 + \phi) \qquad . \qquad (15.a)$$

The force is acting along the longitudinal direction and has real values, $\bar{n}$ is the mean number of photons.

- **The dimensionless force**

We can divide both sides of equation (14.1) by $\hbar k\;\gamma$, to get its dimensionless form;

$$f = -W_1 \qquad (15.b)$$

Where f is the light Pressure force acting along the negative z-direction, which causes a deceleration or accelerating effects on the atoms.
As is well-known , the probability of emission of a photon in a given direction is determined by the intensity of the spherical wave emitted by the quantum nature of

the emission leads to fluctuation of the light-pressure force about the mean value of W_1.

Velocity of atoms after interaction

We shall begin by integrating the force in (15.a)

$$\frac{F}{M} = A = \frac{dv}{dt}$$

$$Since\ , F = -\frac{\hbar k\, G\gamma \bar{n}}{1+G\bar{n}}\cos 2(\theta_1+\phi) = MA$$

$$Then\ , \frac{dv}{dt} = -\frac{\hbar k\, G\gamma \bar{n}}{M(1+G\bar{n})}\cos 2(\theta_1+\phi) =$$

$$\frac{dv}{dt} = -\frac{v_r\, G\gamma \bar{n}}{(1+G\bar{n})}\cos 2(\theta_1+\phi)\ ,\ v_r = \frac{\hbar k}{M}, v_r \rightarrow Recoil\, velocity$$

$$\int_{v_0}^{v} dv = -\frac{v_r\, G\gamma \bar{n}}{(1+G\bar{n})}\int_{t_0}^{t}\cos 2(\theta_1+\phi)\, dt, \theta_1 = \Delta t - kz$$

$$v(t) = v_0 - \frac{v_r\, G\bar{n}}{2\bar{\Delta}(1+G\bar{n})}\{\sin 2(\theta_1+\phi) - \sin 2(\theta_0+\phi)\} \tag{16.a}$$

here , $\theta_0 = \Delta\,\tau_0$, $\bar{\Delta} = \dfrac{\Delta}{\gamma}$.

- **The relative velocity difference:**

$$V = \frac{v(t)-v_0}{v_0} = -\frac{v_r\, G\bar{n}}{2\bar{\Delta}\ v_0(1+G\bar{n})}\{\sin 2(\theta_1+\phi) - \sin 2(\theta_0+\phi)\} \tag{16.b}$$

The temperature of atoms

In a one dimensional space the kinetic energy $E_k = \frac{1}{2}Mv^2$ in the classical mechanics, is equivalent to $E_k = \frac{1}{2}k_B T$ in thermodynamics, so that in the (-ve) z-direction the temperature of atoms will be

$$T(t) = \frac{M}{k_B}v^2 = \frac{M}{k_B}\{v_0 - \frac{v_r\, G\bar{n}}{2\bar{\Delta}(1+G\bar{n})}\{\sin 2(\bar{\theta}_1+\phi) - \sin 2(\theta_0+\phi)\}^2 \tag{17.a}$$

- **The relative temperature difference:**

$$T = \frac{T(t)}{(\frac{Mv_0^{\,2}}{k_B})} = \{1 - \frac{v_r\, G\bar{n}}{2\bar{\Delta}\ v_0(1+G\bar{n})}\{\sin 2(\bar{\theta}_1+\phi) - \sin 2(\theta_0+\phi)\}^2 \quad . \tag{17.b}$$

See table (1) , and Fig. (1) .

The second case

The traveling wave propagates in the $(+ve)$ z-direction ($\vec{K} = k\,\vec{e}_z$). The density matrix elements have the same form but with the new angle $\theta_2 = \Delta t + kz : \Delta > 0$.

$$\hat{\rho}_{11} = \frac{1+0.5G\,\hat{a}^{\dagger}\hat{a}}{1+G\,\bar{n}}\ ,\ \hat{\rho}_{22} = \frac{0.5G\,\hat{a}^{\dagger}\hat{a}}{1+G\,\bar{n}},\ \hat{\rho}_{12} = \frac{g\,\hat{a}^{\dagger}\,e^{-i\theta_2}}{\gamma(1+G\bar{n})}\ ,\ \hat{\rho}_{21} = \frac{g\,\hat{a}\,e^{i\theta_2}}{\gamma(1+G\bar{n})} \tag{18}$$

The force acting on atoms

The Hamiltonian in matrix representation is

$$\hat{H}_I = \begin{bmatrix} 0 & i\hbar g\,\hat{a}\,e^{i\theta_2} \\ -i\hbar g\,\hat{a}^{\dagger}\,e^{-i\theta_2} & 0 \end{bmatrix} \quad ,\theta_2 = \Delta t + kz$$

the induced light pressure force

$$F = \hbar k \gamma\, W_2 \ , \quad W_2 = \frac{G\bar{n}}{1+G\bar{n}}\cos 2(\theta_2 + \phi) \qquad (19.a)$$

in dimensionless forme

$$f = W_2 \ , \qquad (19.b)$$

acting along positive z-direction, which causes a deceleration or acceleration effect on the atoms for chosen different phase angles.

The velocity of atoms;

$$when\ ,\ \frac{dv}{dt} = \frac{\hbar k\, G\gamma\bar{n}}{M(1+G\bar{n})}\cos 2(\theta_2+\phi)$$

$$\frac{dv}{dt} = \frac{v_r\, G\gamma\bar{n}}{(1+G\bar{n})}\cos 2(\theta_2+\phi)\ ,\ v_r = \frac{\hbar k}{M}, v_r \to Recoil\,velocity$$

$$\int_{v_0}^{v} dv = \frac{v_r\, G\gamma\bar{n}}{(1+G\bar{n})}\int_{t_0}^{t}\cos 2(\theta_2+\phi)\,dt,\ \theta_2 = \Delta t + kz$$

$$v(t) = v_0 + \frac{v_r\, G\bar{n}}{2\bar{\Delta}(1+G\bar{n})}\{\sin 2(\theta_2+\phi) - \sin 2(\theta_0+\phi)\} \qquad (20.a)$$

- **The relative velocity difference:**

$$V = \frac{v_r\, G\bar{n}}{2\bar{\Delta}\ v_0(1+G\bar{n})}\{\sin 2(\bar{\theta}_2+\phi) - \sin 2(\theta_0+\phi)\} \qquad (20.b)$$

Since $\bar{\theta}_2 = \bar{\Delta}\tau + 2\pi Z \quad ,\theta_0 = \bar{\Delta}\tau_0$

The temperature of atoms

Similar to the first case:

$$T = \frac{M}{k_B}v^2 = \frac{M}{k_B}\{v_0 + \frac{v_r\, G\bar{n}}{2\bar{\Delta}(1+G\bar{n})}\{\sin 2(\bar{\theta}_2+\phi) - \sin 2(\theta_0+\phi)\}^2 \qquad (21.a)$$

The relative temperature difference:

$$T = \frac{T(t)-T_0}{(\frac{Mv_0{}^2}{k_B})} = \{\frac{v_r\, G\bar{n}}{2\bar{\Delta}\ v_0(1+G\bar{n})}\{\sin 2(\bar{\theta}_2+\phi) - \sin 2(\theta_0+\phi)\}^2 \qquad (21.b)$$

See table (1) and Fig. (2) .

The investigation of the problem in terms of time

The idea of coupling between the mean number of photons and time was introduced by Botin and Kasantsev **[2] and** Pusep **[3];** they adopted that the distribution of the absorbed photons *n* is well approximated as Poisson or Gaussian, according to whether the spontaneous emission is slow or rapid. In the first, the emitted mean photon numbers $\bar{n}$ gives rise also to be Gaussian, it is linearly proportional to time such as $\bar{n} = 0.5\gamma t$, *and* $\tau = \gamma t$, it is known also as strong field. For rapid spontaneous emission n is Gaussian and the mean photon numbers $\bar{n}$ is Gaussian as well, where $\bar{n} = 2G\tau$, it is known also as weak field $G << 1$. Here t is the interaction time, while for slow spontaneous emission $G \gg 1$.

First case:

The traveling wave in (-ve) z-direction and , $\Delta > 0$. The density matrix elements are:

$$\hat{\rho}_{11} = \frac{1+0.5G\hat{a}^{\dagger}\hat{a}}{1+0.5G\tau} \;,\hat{\rho}_{22} = \frac{0.5G\hat{a}^{\dagger}\hat{a}}{1+0.5G\tau},\; \hat{\rho}_{12} = \frac{g\,\hat{a}^{\dagger}\,e^{-i\theta_1}}{\gamma(1+0.5G\tau)} \;,\; \hat{\rho}_{21} = \frac{g\,\hat{a}\,e^{i\theta_1}}{\gamma(1+0.5G\tau)} \tag{22}$$

Force acting on atoms

Force using photons number in terms of time representation with strong field

$$f = -\frac{G\tau}{2+G\tau}\cos 2(\bar{\theta}_1 + \phi) \tag{23.a}$$

Velocity of atoms after interaction

Velocity using photons number in terms of time representation with strong field

$$V = -\frac{v_r\, G}{v_0}\int_{\tau_0}^{\tau} \frac{\tau}{(2+G\tau)}\cos 2(\bar{\theta}_1 + \phi)\,d\tau \tag{23.b}$$

Temperature of atoms

Temperature using photons number in terms of time representation with strong field

$$T = \{1 - \frac{v_r\, G}{v_0}\int_{\tau_0}^{\tau} \frac{\tau}{(2+G\tau)}\cos 2(\bar{\theta}_1 + \phi)\,d\tau\}^2 \tag{23.c}$$

Second case:

The traveling wave in (+ve) z-direction and , $\Delta > 0$. The density matrix elements are:

$$\hat{\rho}_{11} = \frac{1+0.5G\hat{a}^{\dagger}\hat{a}}{1+0.5G\tau} \;,\hat{\rho}_{22} = \frac{0.5G\hat{a}^{\dagger}\hat{a}}{1+0.5G\tau} \;,\hat{\rho}_{12} = \frac{g\,\hat{a}^{\dagger}\,e^{-i\theta_2}}{\gamma(1+0.5G\tau)} \;,\; \hat{\rho}_{21} = \frac{g\,\hat{a}\,e^{i\theta_2}}{\gamma(1+0.5G\tau)} \tag{24}$$

Force acting on atoms

Force using photons number in terms of time representation with strong field

$$f = \frac{G\tau}{2+G\tau}\cos 2(\bar{\theta}_2 + \phi) \tag{25.a}$$

Velocity of atoms after interaction

Velocity using photons number in terms of time representation with strong field

$$V = \frac{v_r\, G}{v_0}\int_{\tau_0}^{\tau} \frac{\tau}{(2+G\tau)}\cos 2(\bar{\theta}_2 + \phi)\,d\tau \tag{25.b}$$

Temperature of atoms

Temperature using photons number in terms of time representation with strong field

$$T = \{1 - \frac{v_r\ G}{v_0} \int_{\tau_0}^{\tau} \frac{\tau}{(2 + G\tau)} \cos 2(\overline{\theta}_2 + \phi)\, d\tau\}^2 \qquad (25.c)$$

See table (2) and Fig. (3) and (4).

A. Rapid spontaneous emission (weak field $\frac{g}{\gamma}$ <<1)

First case:

the traveling wave in (- ve) z-direction and , $\Delta > 0$. The density matrix elements are**:**

$$\hat{\rho}_{11} = \frac{1 + 0.5\,G\,\hat{a}^{\dagger}\hat{a}}{1 + 2\,G^2\,\tau},\ \hat{\rho}_{22} = \frac{0.5\,G\,\hat{a}^{\dagger}\hat{a}}{1 + 2\,G^2\,\tau},\ \hat{\rho}_{12} = \frac{g\,\hat{a}^{\dagger}\,e^{-i\theta_1}}{\gamma(1 + 2\,G^2\,\tau)},\ \hat{\rho}_{21} = \frac{g\,\hat{a}\,e^{i\theta_1}}{\gamma(1 + 2\,G^2\,\tau)} \qquad (26)$$

Force acting on atoms

We express the force using photons number in terms of time representation with weak field

$$f = -\frac{2\,G^2\,\tau}{1 + 2G^2\,\tau} \cos 2(\overline{\theta}1 + \phi) \qquad (27.a)$$

Velocity of atoms after interaction

Velocity using photons number in terms of time representation with weak field

$$V = -\frac{2v_r\ G^2}{v_0} \int_{\tau_0}^{\tau} \frac{\tau}{(1 + 2G^2\,\tau)} \cos 2(\overline{\theta}1 + \phi)\, d\tau \qquad (27.b)$$

Temperature of atoms

Temperature using photons number in terms of time representation with weak field

$$T = \{1 - \frac{2v_r\ G^2}{v_0} \int_{\tau_0}^{\tau} \frac{\tau}{(1 + 2G^2\,\tau)} \cos 2(\overline{\theta}1 + \phi)\, d\tau\}^2 \qquad (29.c)$$

Second case :

the traveling wave in (+ve) z-direction and , $\Delta > 0$.The density matrix elements are**:**

$$\hat{\rho}_{11} = \frac{1 + 0.5\,G\,\hat{a}^{\dagger}\hat{a}}{1 + 2\,G^2\,\tau},\ \hat{\rho}_{22} = \frac{0.5\,G\,\hat{a}^{\dagger}\hat{a}}{1 + 2\,G^2\,\tau},\ \hat{\rho}_{12} = \frac{g\,\hat{a}^{\dagger}\,e^{-i\theta_2}}{\gamma(1 + 2\,G^2\,\tau)},\ \hat{\rho}_{21} = \frac{g\,\hat{a}\,e^{i\theta_2}}{\gamma(1 + 2\,G^2\,\tau)} \qquad (30)$$

Force acting on atoms

Force using photons number in terms of time representation with weak field

$$f = \frac{2G^2\,\tau}{1 + 2G^2\,\tau} \cos 2(\overline{\theta}_2 + \phi) \qquad (31.a)$$

Velocity of atoms after interaction

Velocity using photons number in terms of time representation with weak field

$$V = \frac{2v_r\ G^2}{v_0} \int_{\tau_0}^{\tau} \frac{\tau}{(1 + 2G^2\,\tau)} \cos 2(\overline{\theta}_2 + \phi)\, d\tau \qquad (31.b)$$

Temperature of atoms

Temperature using photons number in terms of time representation with weak field

$$T=\{1+\frac{2v_r\ G^2}{v_0}\int_{\tau_0}^{\tau}\frac{\tau}{(1+2G^2\,\tau)}\cos 2(\overline{\theta}\,2+\phi)\,d\tau\}^2 \qquad (31.c)$$

See table (3) , and Fig. (5) and (6)

Irreversible statistical dynamics of the problem

Entropy of the system

The most natural measure of the uncertainty of the quantum- mechanical state is the entropy. The quantum Mechanical Entropy due to Von-Neumann reads **[18]**

$$S_{\hat{\rho}}=-k_B\,Tr\{\hat{\rho}\ln\hat{\rho}\}\quad,\hat{\rho}_{2x\,2} \qquad (32.a)$$

Where $\hat{\rho}$ is the density operator of the quantum mechanical system with k_B is the Boltzmann's constant. The Von-Neumann entropy of a pure state is equal to zero. For Quantum mechanical mixture the Von-Neumann entropy is larger than zero.
By deriving the eigenvalues of density matrix **[19]**

$$S_{\hat{\rho}}=-k_B\sum_{l=1}^{d}\lambda_l\log\lambda_l=-k_B\,\{\lambda_1\log\lambda_l+\lambda_2\log\lambda_2\} \qquad (32.b)$$

to identify an eigenvalues of the density matrix from the eigen equation

$$\begin{vmatrix}\overline{\rho}_{11}-\lambda & \overline{\rho}_{12}\\ \overline{\rho}_{21} & \overline{\rho}_{22}-\lambda\end{vmatrix}=0\ \Rightarrow\lambda^2-\lambda+\overline{\rho}_{11}\overline{\rho}_{22}-\overline{\rho}_{12}\overline{\rho}_{21}=0\,.$$

Using eqs. (10.1), (10.2), (10.3) and (10.4), we get:

$$\lambda_{1,2}=\frac{1}{2}\pm\frac{\sqrt{1-2G\,\overline{n}}}{2(1+G\,\overline{n})} \qquad (32.c)$$

According to**[20]** the stability of the system is described according to the eigenvalues in the form

$\lambda_1=\overline{\lambda}_1=a+i\,b\,:a>0\ and\quad b>0$.

So that the system behaves as a " unstable spiral " meaning that the atoms dissipate /acquires energy to/ from the laser field as a energy bath described by the variables mean photon numbers $\overline{n}$. Therefore;

$$S_{\hat{\rho}}=-k_B\,\{\lambda_1\log\lambda_l+\lambda_2\log\lambda_2\} \qquad (32.d)$$

The dimensionless entropy:

Entropy has the dimension of Boltzmann constant, its dimensionless form will be;

$$S=\frac{S_{\hat{\rho}}}{k_B}=-\{\lambda_1\log\lambda_l+\lambda_2\log\lambda_2\}\quad . \qquad (32.e)$$

Here S is time independent, as a result the entropy production $\dot{S}$, could be calculated in either case of strong and weak fields.
the illustrations are seen in Fig. (7)

B: For slow spontaneous emission (strong field):

Entropy of the system

The eigenvalues in strong field are

$$\lambda_{1,2} = \frac{1}{2} \pm \frac{\sqrt{1 - G\tau}}{(2 + G\tau)} \qquad (33.a)$$

Then ,dimensionless entropy in strong field

$$S_{str} = -\{\lambda_1 \log \lambda_l + \lambda_2 \log \lambda_2\} \;\; , \;\; \lambda_{1,2} = \lambda_{1,2}(t) \qquad (33.b)$$

Entropy production

Resorting to Kasantsev and Pusep who adopted the notion of strong and weak fields, which permit to investigate the entropy production.

In the conventional expression of the field in terms of the photon numbers, **S** is independent of time , as a result the entropy production could not be evaluated, while at strong and weak fields, the eigenvalues depend on time, therefore

$$\sigma_{st} = \frac{d}{d\tau} S_{st} \qquad (34.a)$$

$$\sigma_{st} = \frac{G\left[(-2 + 0.5\,G\tau) Log\left[0.5 - \frac{\sqrt{1-G\tau}}{(2+G\tau)}\right] + (2 - 0.5\,G\tau) Log\left[0.5 + \frac{\sqrt{1-G\tau}}{(2+G\tau)}\right]\right]}{\sqrt{1-G\tau}\,(2+G\tau)^2} \qquad (34.b)$$

See Fig.(8).

<u>For strong spontaneous emission (weak field) :</u>

Entropy of the system

The eigenvalues in weak field is ;

$$\lambda_{1,2} = \frac{1}{2} \pm \frac{\sqrt{1 - 2G^2\tau}}{2(1 + 2G^2\tau)} \qquad (35.a)$$

Then , dimensionless entropy in strong field is;

$$S_w = -\{\lambda_1 \log \lambda_l + \lambda_2 \log \lambda_2\} \qquad (35.b)$$

Here S is time dependent, as a result the entropy production $\dot{S} = \sigma$, could be calculated.

Entropy production

$$\sigma_w = \frac{d}{d\tau} S_w \qquad (36.a)$$

$$\sigma_w = \frac{G^2\left[(-1.5 + G^2\tau) Log\left[0.5 - \frac{\sqrt{1-2G^2\tau}}{(2+4G^2\tau)}\right] + (1.5 - G^2\tau) Log\left[0.5 + \frac{\sqrt{1-2G^2\tau}}{(2+4G^2\tau)}\right]\right]}{\sqrt{1-2G^2\tau}\,(1+2G^2\tau)^2} ,\qquad (36.b)$$

See Fig.(9)

Table (1)

The first case the traveling wave in (-ve) z-direction and , $\Delta > 0$					
Behavior	ϕ	θ_1	**Behavior**	ϕ	θ_1
Acceler- Ation	$\pi/6$	$\pi/3$	**Deceler- ation**	$\pi/2$	$2\pi/3$
	$\pi/3$	$\pi/6$		$2\pi/3$	$\pi/3$
	$\pi/2$	$\pi/6$		$5\pi/6$	$\pi/3$
The second case the traveling wave in (+ve) z-direction and , $\Delta > 0$					
Behavior	ϕ	θ_2	**Behavior**	ϕ	θ_2
Acceler- Ation	$\pi/2$	$2\pi/3$	**Deceler- ation**	$\pi/6$	$\pi/3$
	$2\pi/3$	$\pi/3$		$\pi/3$	$\pi/6$
	$5\pi/6$	$\pi/6$		$\pi/2$	$\pi/6$

Table (2) : At strong field

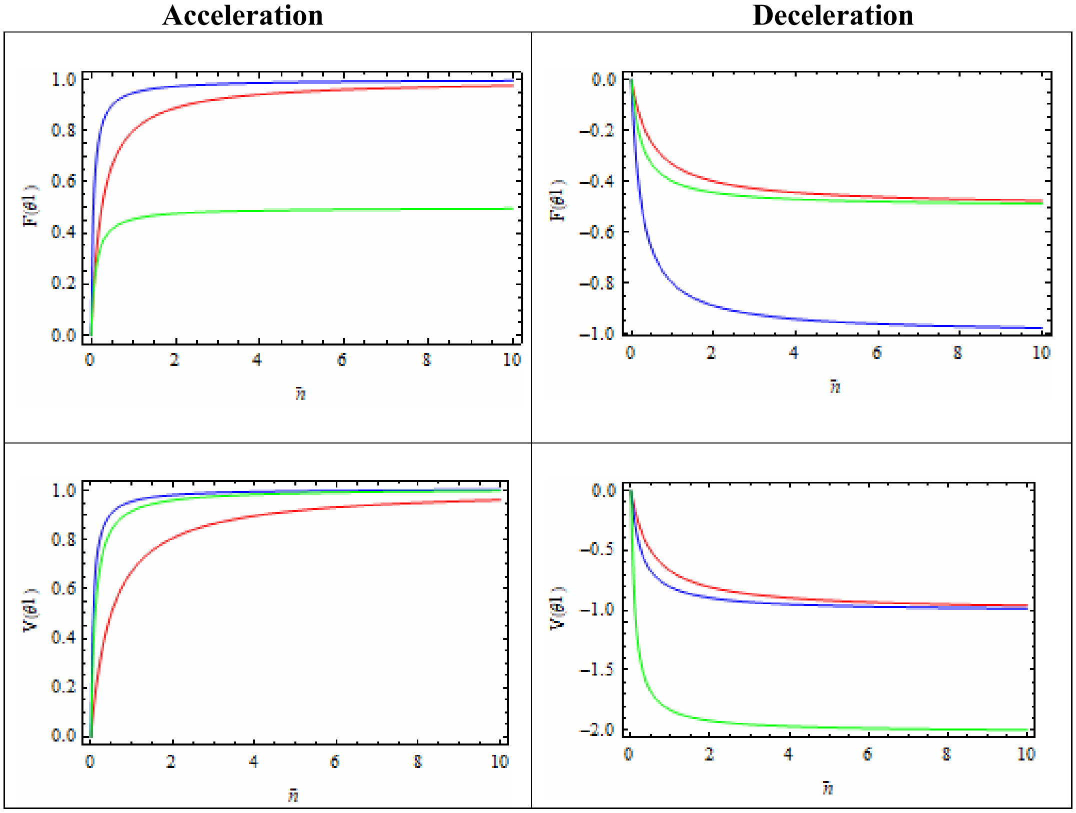
The first case					
Behavior	ϕ	θ_1	**Behavior**	ϕ	θ_1
Acceleration	$\pi/6$	$\pi/2$	**Deceleration**	$\pi/3$	$5\pi/6$
	$\pi/2$	$\pi/6$		$\pi/2$	$2\pi/3$
	$\pi/3$	$\pi/3$		$2\pi/3$	$\pi/2$
	$5\pi/6$	$5\pi/6$		$5\pi/6$	$\pi/3$
The second case					
Behavior	ϕ	θ_2	**Behavior**	ϕ	θ_2
Acceleration	$\pi/3$	$5\pi/6$	**Deceleration**	$\pi/6$	$\pi/2$
	$\pi/2$	$2\pi/3$		$\pi/2$	$\pi/6$
	$2\pi/3$	$\pi/2$		$\pi/3$	$\pi/3$
	$5\pi/6$	$\pi/3$		$5\pi/6$	$5\pi/6$

Table (3) : At weak field

The first case					
Behavior	ϕ	θ_1	**Behavior**	ϕ	θ_1
Acceleration	$\pi/6$	$\pi/6$	**Deceleration**	$\pi/2$	$2\pi/3$
	$\pi/2$	$5\pi/6$		$\pi/3$	$\pi/2$
	$2\pi/3$	$2\pi/3$		$\pi/2$	$\pi/3$
	$5\pi/6$	$\pi/2$		$2\pi/3$	$\pi/6$
The second case					
Behavior	ϕ	θ_2	**Behavior**	ϕ	θ_2
Acceleration	$\pi/3$	$5\pi/6$	**Deceleration**	$\pi/6$	$\pi/2$
	$\pi/2$	$2\pi/3$		$\pi/3$	$\pi/3$
	$2\pi/3$	$\pi/2$		$\pi/2$	$\pi/6$
	$5\pi/6$	$\pi/3$		$5\pi/6$	$5\pi/6$

The first case: the traveling wave in (-ve) z-direction and , $\Delta > 0$.

Acceleration **Deceleration**



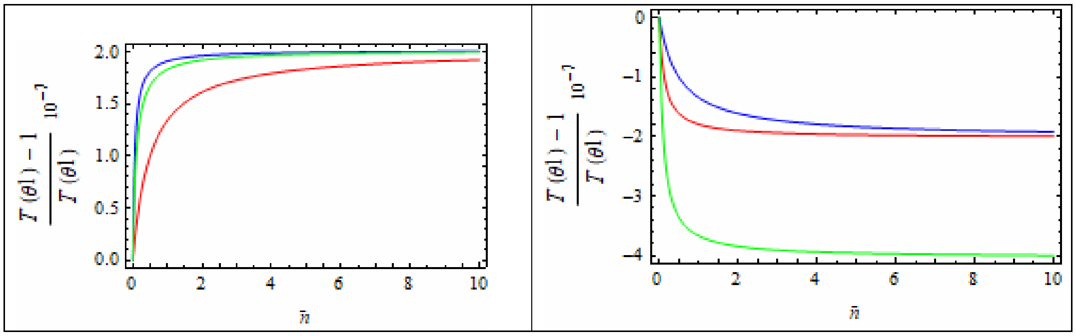


Fig. (1)

The second case: the traveling wave in (+ve) z-direction and , $\Delta > 0$.

Acceleration **Deceleration**

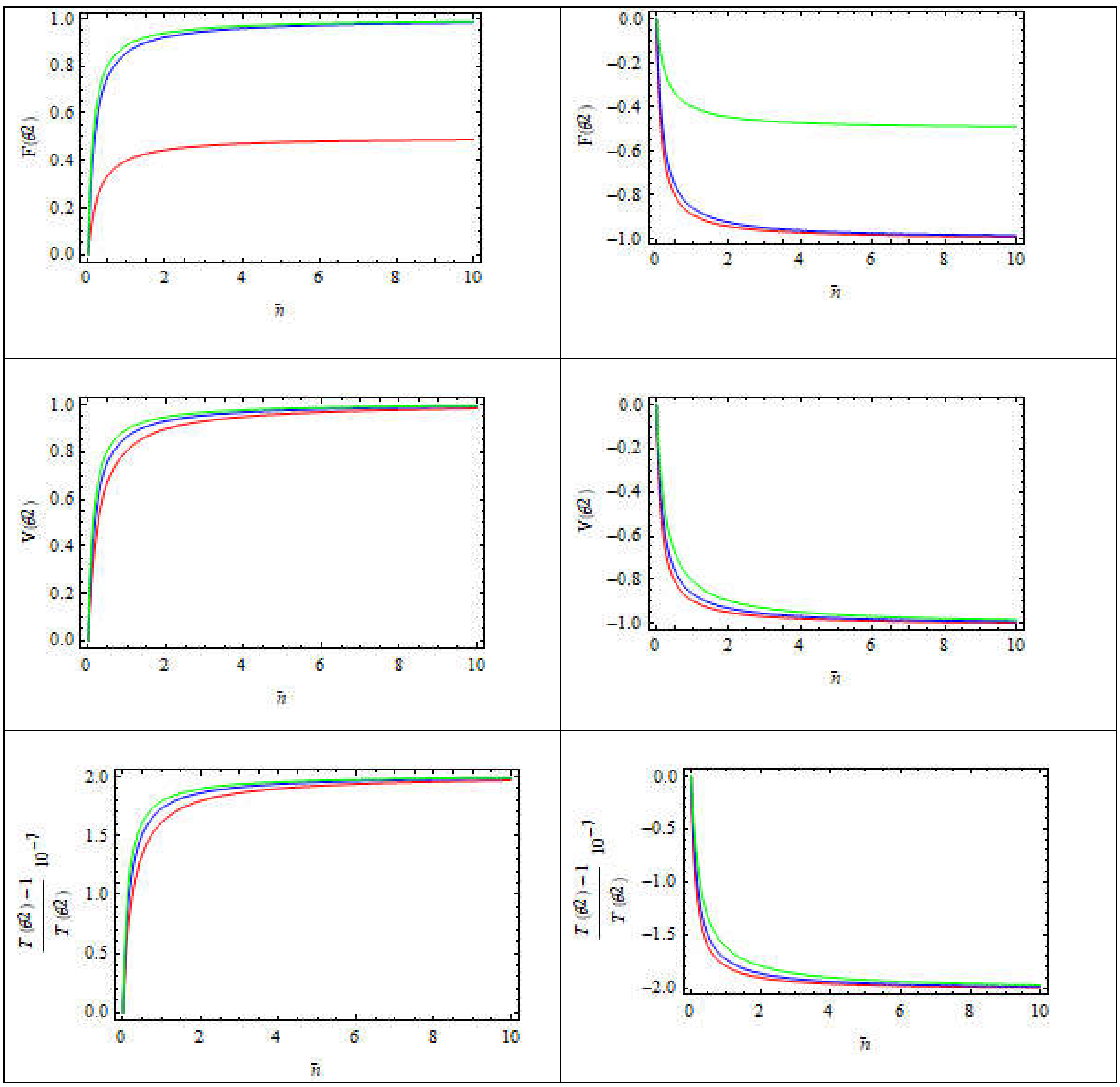


Fig.(2)

The first :

$\Delta > 0$, *and* $\vec{K}_{st} = -k\,\vec{e}_z$

Deceleration **Acceleration**

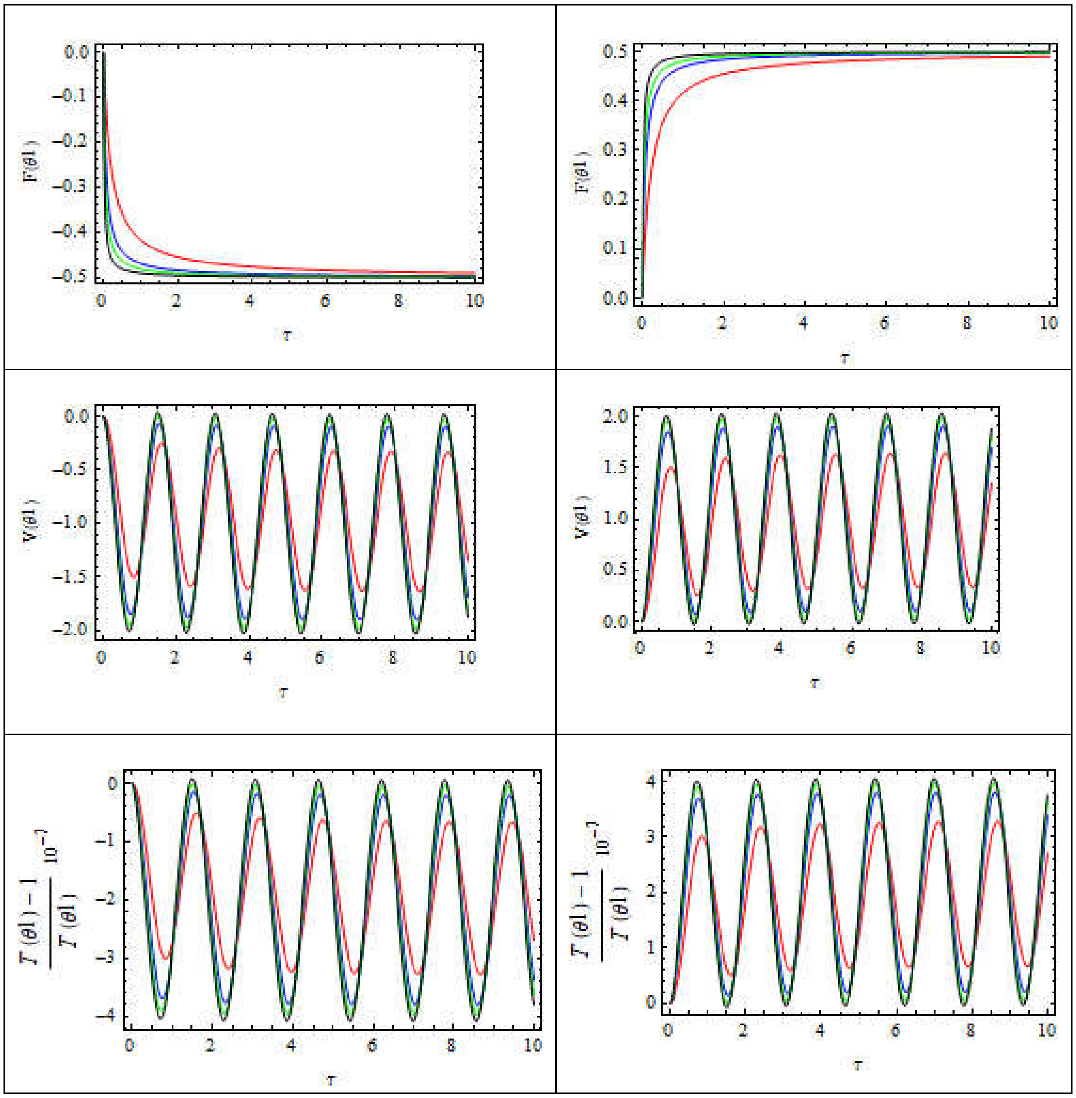


Fig.(3)

<u>The second</u> : $\Delta > 0$, *and* $\vec{K}_{st} = k\,\vec{e}_z$

<u>Deceleration</u> **<u>Acceleration</u>**

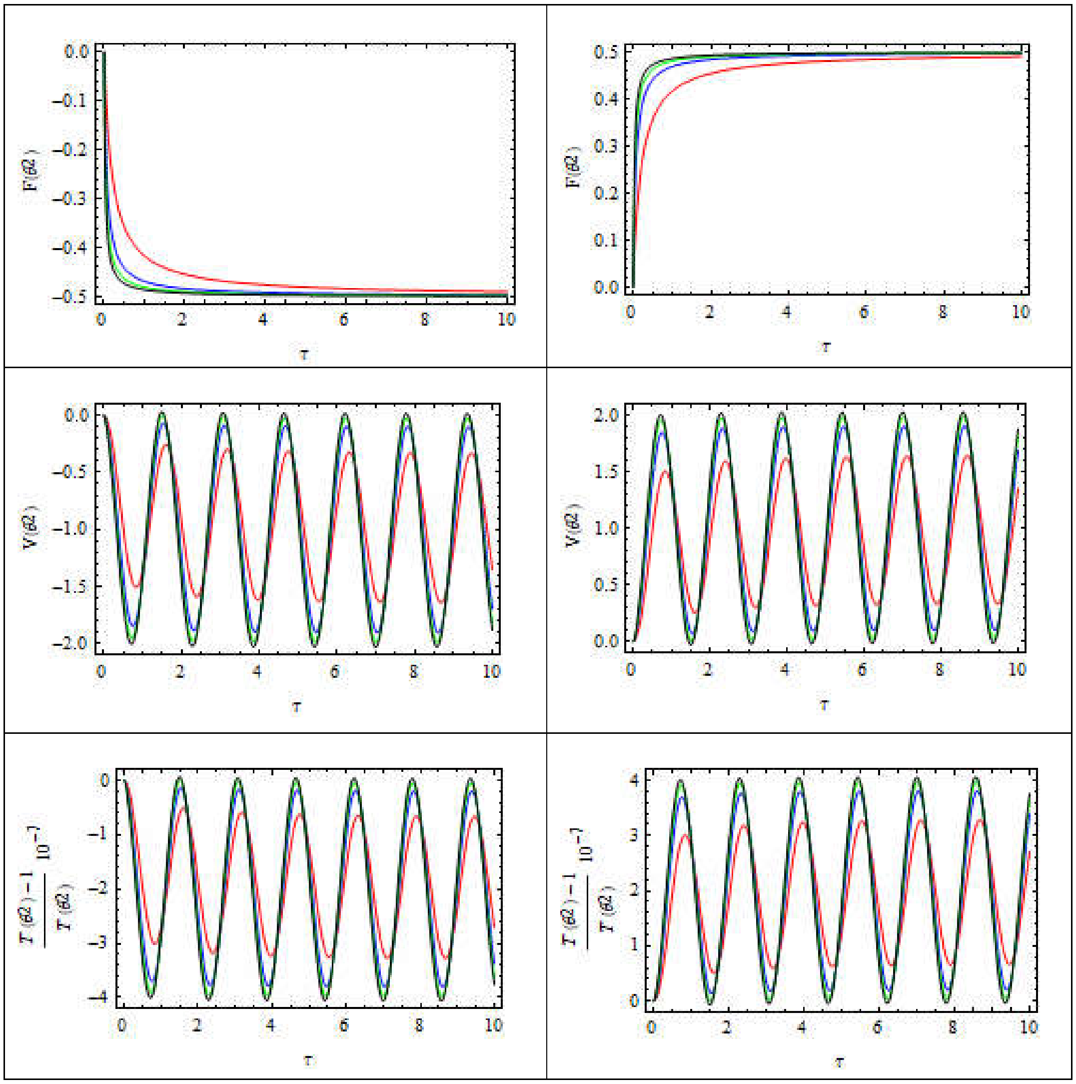


Fig.(4)

The first :

$\Delta > 0$ *, and* $K_w = -k\, e_z$

Deceleration **Acceleration**

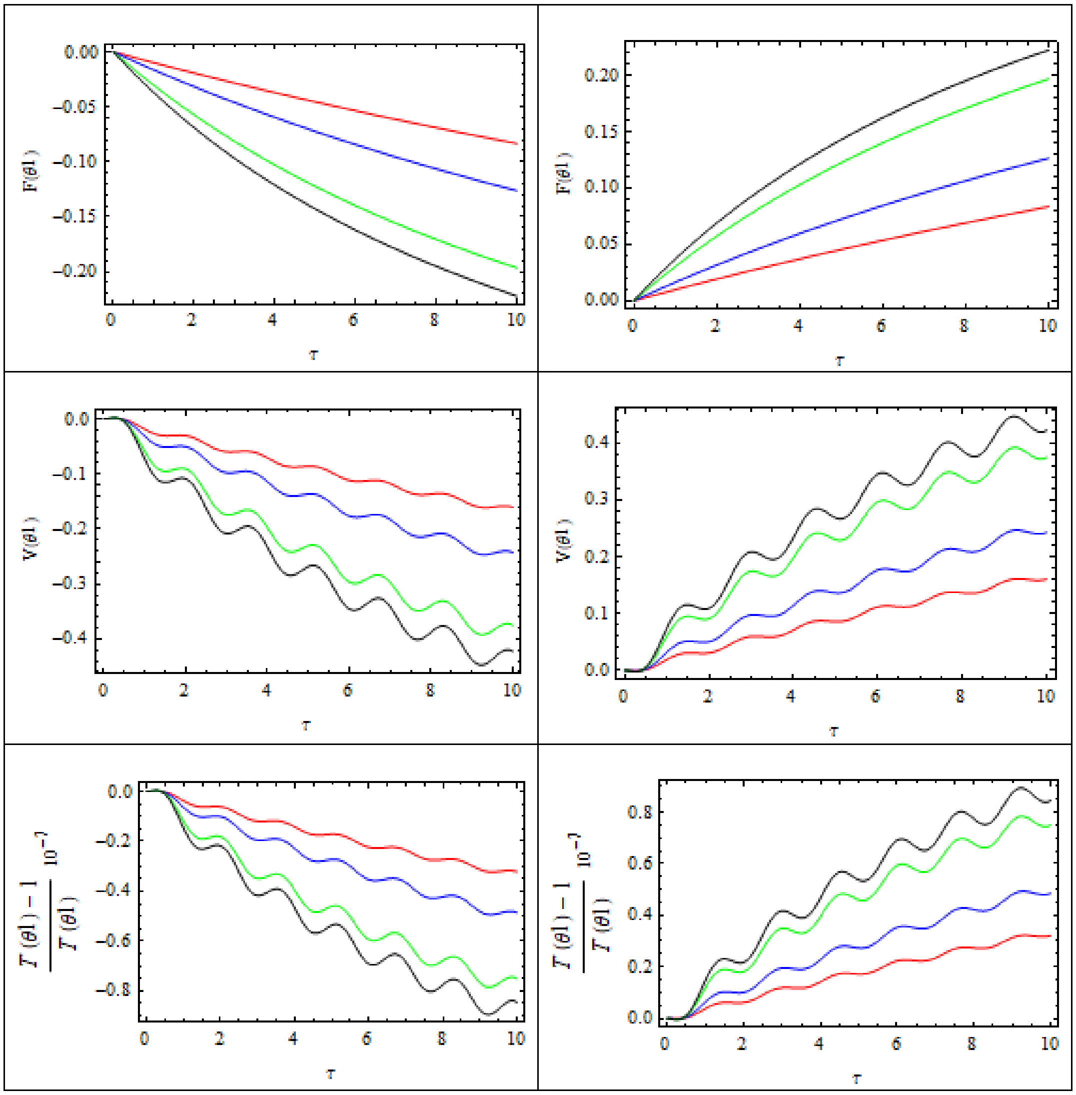


Fig.(5)

The second case : $\Delta > 0$ *, and* $K_w = k\, e_z$

Deceleration **Acceleration**

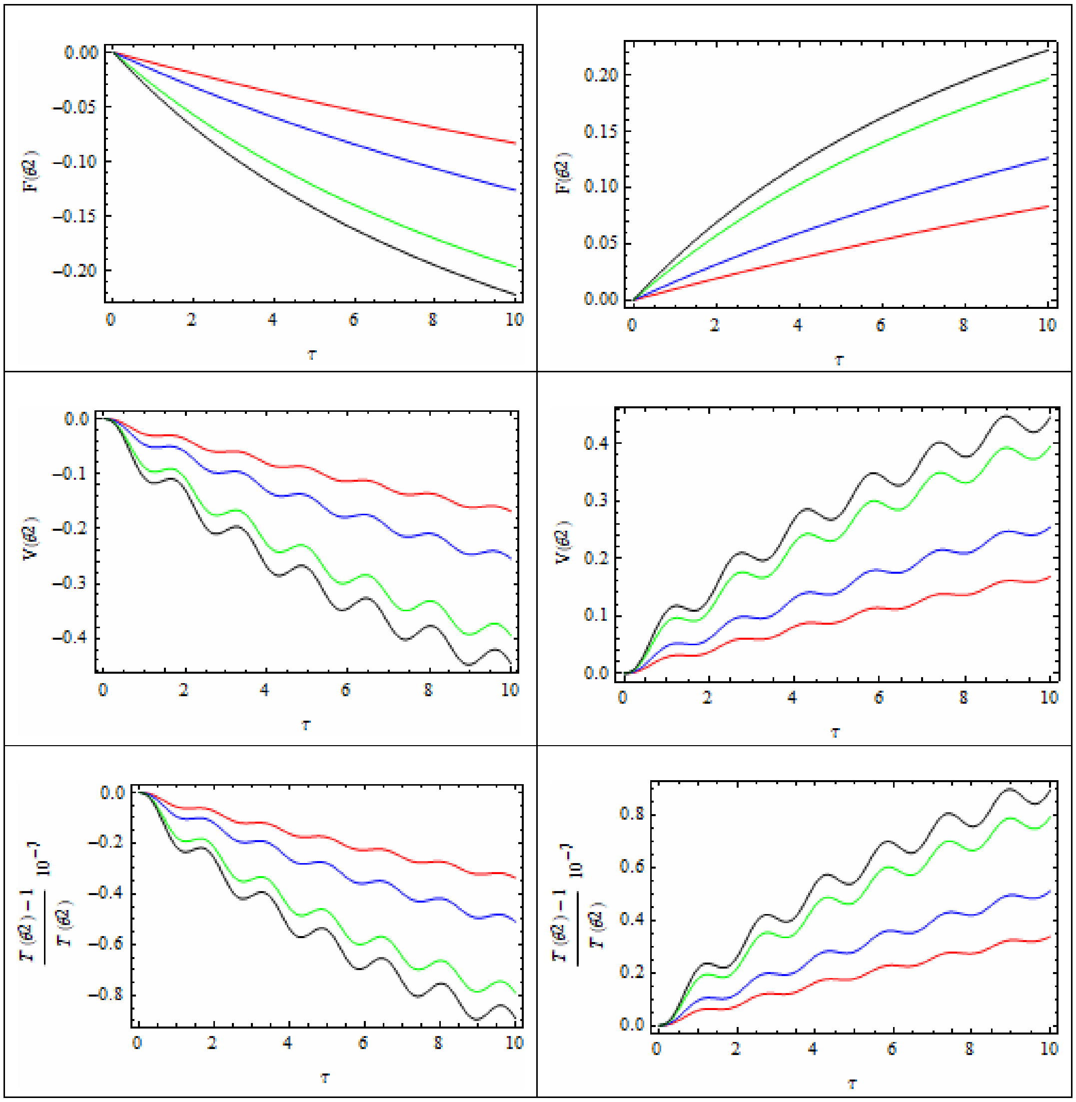


Fig.(6)

Entropy and entropy production of the system:

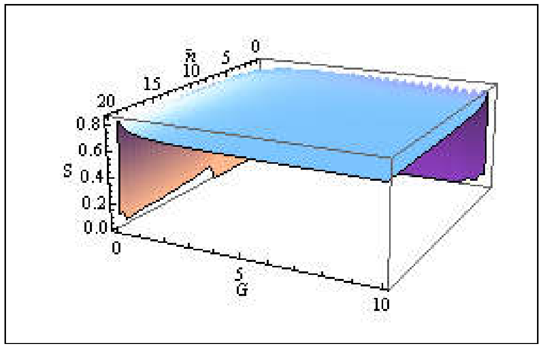


Fig. (7)

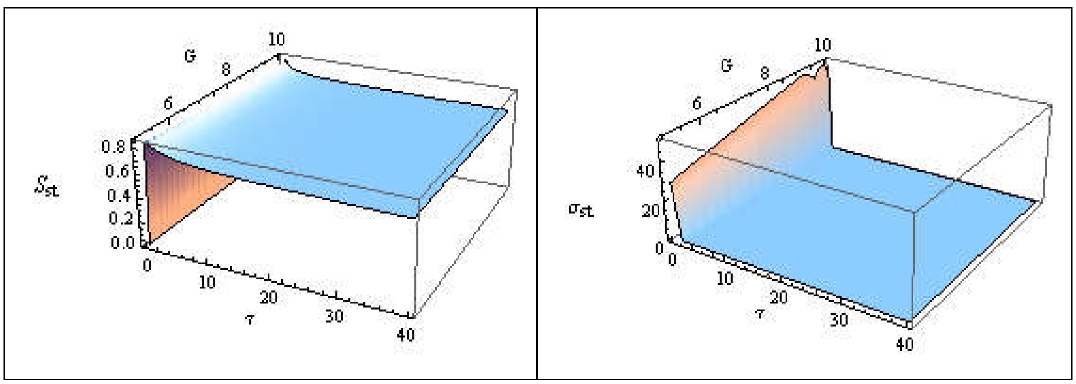


Fig. (8)

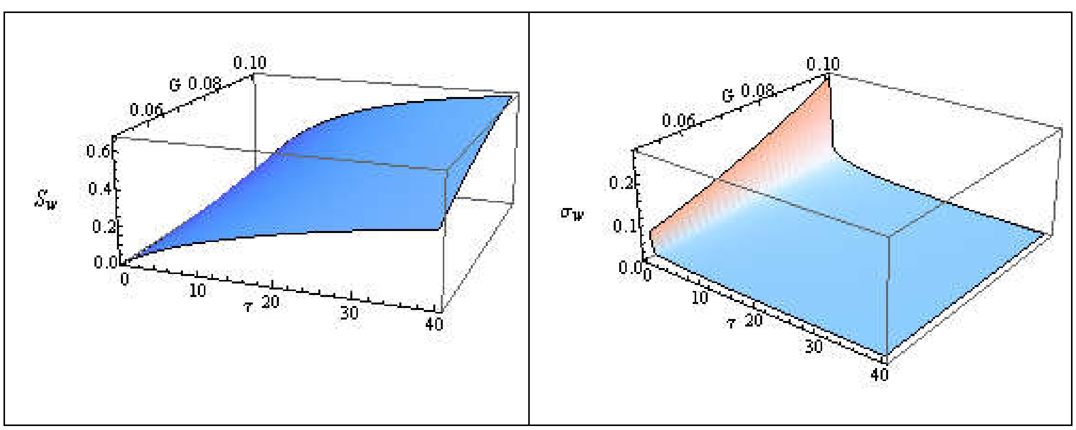


Fig. (9)

Discussion and conclusion

- We deal with the photon number taken as a continuous variable according to **[21, 22]**. The study is conducted using sodium atoms vapor **[23].**
- Taking into consideration that the collision between atoms is neglected.
- The steady state of the optical Bloch equations for the density matrix elements are evaluated, then the mean force with respect to the coherent state causes the acceleration and deceleration and also the change in detuning, taking in details the first two out of four cases:

1) $\vec{K} = -k\,\vec{e}_z\,,\ \Delta > 0$ 2) $\vec{K} = k\,\vec{e}_z\,,\ \Delta > 0$

3) $\vec{K} = -k\,\vec{e}_z\,,\ \Delta < 0$ 4) $\vec{K} = k\,\vec{e}_z\,,\ \Delta < 0$

- The evolution in the behavior of atoms is investigated according to changing the angles of coherence through applying optical filters, playing a principal role in inducing deceleration or acceleration of atoms.

It is kept in mind that, the atomic vapor is immersed in an energy bath presented by the laser field, such that the state the atomic vapor is unstable inside the system due to the loss or gain of energy caused by the deceleration or acceleration of atoms influenced by the change of coherence angles.

The investigation of the problem according to Kazanstev and Pusep in terms of time reveals that in the case of:

a) Slow spontaneous emission (strong field):

The mean number of photons in the time representation is $\bar{n} = 0.5\,\tau$, where the field is strong and the saturation coefficient is taken such that $G >> 1$. The proper choice of coherence angles also control the behavior of the atoms, which are shown in the table (3) and also in Figs. (3) and (4).

b) Rapid spontaneous emission (weak field):

The mean number of photons was treated such that $\bar{n} = 2\,G\,\tau$, where the field is weak i.e. $G << 1$. The coherence angles also control the behavior of the atoms, which are shown in the table (4) and Figs. (5) and (6).

When studying the state of the system by evaluating the entropy in the classical representation of the field, Fig. (7) explains that the system behaves as " unstable spiral ".

Throughout the investigation of the problem, in terms of mean photon numbers it is been found that**:** in the classical approach the mean number of photons is dealt with as a continuous quantity by letting it changes within the interval [0,20]. While in terms of time, comparing both slow and rapid spontaneous emission cases, regardless of the coherence angles; the entropy production behaves such that it suddenly increases with increasing G at the beginning of the time interval, then drops severely to zero during the rest of the duration approaching the state of equilibrium with the increase in entropy, where the energy is dissipated to the field, in agreement with the notion of open systems [8], As shown in Figs. (8) and (9).

- This study is a revisiting and verification of the previous work in [4], which deserves a thorough revision and deeper insight into the evolution of the system under study.

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